

Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

L. H. DONNELL, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

I. MECHANICS OF SOLIDS

1. *Dynamics*

General Kinematics, Statics, Dynamics	25
Gyroscopics, Governors, Servos	25
Vibrations, Balancing	26
Wave Motion, Impact	26

2. *Primarily Elastic Behavior*

Elasticity Theory	27
Rods, Beams, Shafts, Springs, Cables, etc.	28
Plates, Disks, Shells, Membranes	29
Buckling Problems	29
Structures.	30

3. *Primarily Nonelastic Behavior*

Rheology (Plastic, Viscoplastic Flow)	30
Failure, Mechanics of Solid State	31
Design Factors, Meaning of Material Tests	31
Material Test Techniques	31
Mechanical Properties of Specific Mate- rials.	32
Mechanics of Forming and Cutting	32

II. MECHANICS OF FLUIDS

Hydraulics; Cavitation; Transport	32
Incompressible Flow: Laminar; Viscous	33
Compressible Flow, Gas Dynamics	35
Turbulence, Boundary Layer, etc.	38
Aerodynamics of Flight; Wind Forces	40
Aeroelasticity (Flutter, Divergence, etc.)	41
Propellers, Fans, Turbines, Pumps, etc.	42
Flow and Flight Test Techniques	43

III. HEAT

Thermodynamics	44
Heat Transfer; Diffusion	45

IV. MISCELLANEOUS

Theoretical and Experimental Methods	46
Acoustics	47
Ballistics, Detonics (Explosions)	47
Soil Mechanics, Seepage	47
Geophysics, Meteorology, Oceanography	48
Marine Engineering Problems	48

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

February 1949

Vol. 2, No. 2

General Kinematics, Statics, Dynamics

(See also Revs. 156, 158, 182, 195, 275, 277)

152. Bruto Caldonazzo, "On free motions of a continuous medium (Sui moti liberi di un mezzo continuo)," *Ann. Mat. pura appl.*, 1947, ser. 4, vol. 26, pp. 43-55.

By a "free" motion of a continuum, the author means one in which the acceleration is everywhere zero, that is, the vector velocity of each particle is constant. The author points out that such motions are not trivial; in particular, they are in general not steady. Let $v_0(r_0)$ be the vector velocity field at time $t = 0$. Then, since at all subsequent times the velocity v is given by the Lagrangian relation $v(r_0, t) = v_0(r_0)$, but $r = r_0 + vt$, to find an Eulerian expression for v we must solve the functional equation $v = v_0(r - vt)$. Conversely, if v_0 be an arbitrary vectorial function of one vectorial variable, this last expression yields a motion with zero acceleration. The author discusses various special cases. For example, he proves that in a free motion in which the speed is steady but not constant in space, the surfaces of equal speed are steady material planes, and he exhibits an example of such a motion. He proves that if ρ be a distribution of density compatible with a free motion, so is $\rho g(v)$, the function g being arbitrary.

C. A. Truesdell, USA

153. A. L. Zelmanov, "The application of deforming co-ordinates in nonrelativistic mechanics" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Aug. 21, 1948, vol. 61, pp. 993-996.

The Lagrangian particle labels x^1, x^2, x^3 can be used as time-dependent curvilinear space co-ordinates, and the $\partial/\partial x$ derivatives can be introduced into the equations of motion in the place of the Eulerian spatial ones. If $ds^2 = h_{ik}x^i x^k$, and $R_{ik} = \partial u_i/\partial x_k - \partial u_k/\partial x_i$, the equation of continuity is $\partial(\partial h^{1/2})/\partial t = 0$, and the equation of motion $\partial R_{ik}/\partial t = \partial S_i/\partial x_k - \partial S_k/\partial x_i$ where S_i is the resultant vector of body forces and stresses per unit mass. If the space is Euclidean, the covariant curl, G_{ijk} , of $R_{ik} + \partial h_{ik}/\partial t$ vanishes. Compatibility conditions of the equations of motion are given in terms of the tensors h, G and the curvature tensor.

A. W. Wundheiler, USA

Gyroscopics, Governors, Servos

(See also Rev. 237)

154. M. R. Hannah, "Frequency-response measurements of a hydraulic power unit," *Trans. Amer. Soc. mech. Engrs.*, July 1948, vol. 70, pp. 525-534.

This paper reports a very comprehensive series of tests to determine the transmission characteristics of a hydraulic servo-unit under constant amplitude, variable frequency, sinusoidal excitation. Response characteristics were determined for individual elements of the system, as well as for the over-all system, and the experimental results are compared with theoretically derived response curves in order to extend the range of the test

data to cover possible improvements of the servo-unit. It was shown that the over-all response of the servo-unit could be reasonably approximated by a linear constant-coefficient differential equation of low order.

Excellent discussions of the measurement techniques used and the fitting of approximate equations to the test data are included.

Henry W. Foster, USA

155. V. V. Solodovnikov, "Criteria for absence of overregulation and criteria for monotony" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Oct. 11, 1948, vol. 62, pp. 599-602.

The paper shows that, the transfer function being

$$x(t) = x_0 + (2/\pi) \int_0^\infty Y(\omega) \cos t\omega d\omega/\omega$$

where $Y(\omega)$ is the imaginary frequency characteristic, the criterion for the prevention of overregulation is that $Y(\omega)/\omega$ be negative.

If

$$x(t) = (2/\pi) \int_0^\infty X(\omega) \sin t\omega d\omega/\omega$$

where $X(\omega)$ is the real frequency characteristic, the criterion for monotony is that $X(\omega)$ be positive.

These criteria are applied to the system

$$\frac{d^2x}{dt^2} + 2\zeta \frac{dx}{dt} + x = [1],$$

where $[1]$ denotes the step function.

Walter W. Soroka, USA

156. Giuseppe Grioli, "Existence and determination of dynamically possible regular precessions for a rigid asymmetric body subject to gravity only (Esistenza e determinazione delle precessioni regolari dinamicamente possibili per un solido pesante asimmetrico)," *Ann. Mat. pura appl.*, 1947, ser. 4, vol. 26, pp. 271-281.

The author considers a solid body S whose central momental ellipsoid E_G has unequal axes. Suppose that a point O of the body, different from the center of gravity G , is fixed by means of a frictionless joint. The author searches for regular undegenerate precessions among the possible movements of the body. He shows that there exist ∞^2 regular precessions which he determines completely as follows: Let D be one of the two straight lines which pass through G and are normal to the planes of circular sections of E_G ; let α be the acute angle between D and the normal to E_G at the intersection point of this straight line with E_G . In the regular precessions obtained the point O must be on D which is the axis of self-rotation; the axis of precession makes the angle α with the vertical and is perpendicular to D ; the period of precession and that of self-rotation are equal.

It is not necessary that S have some particular structural properties in order that one of these regular precessions be possible;

the initial conditions alone determine whether regular precession occurs, provided that the condition regarding the choice of O is satisfied. The author shows that the regular precessions in question are the only ones dynamically possible. If E_G is an ellipsoid of revolution (gyroscope), the author shows that the axis of the only and well-known possible regular precession is vertical, and the axis of self-rotation coincides with the gyroscopic axis. Conversely, this type of precession is possible in the gyroscopic case only.

Ratip Berker, Turkey

Vibrations, Balancing

(See also Revs. 154, 182, 200, 248)

157. Cataldo Agostinelli, "On Cauchy's problem for the differential equation of the vibrating plate (Sul problema di Cauchy per l'equazione differenziale delle piastre vibranti)," *Ann. Mat. pura appl.*, 1947, ser. 4, vol. 26, pp. 27-41.

The general solution of the small vibrations of an infinite homogeneous plane plate with given initial displacement and velocity is developed in the form of a definite integral. The results are extended to plates of finite size, and in particular, as examples, the rectangular plate simply supported around the edges and the circular plate built-in around its circumference are worked out. The method employed is to break up the original differential equation of the fourth order into two second-order equations and to use methods analogous to those used in heat-flow problems.

Edward Saibel, USA

158. I. P. Makarov, "New stability criteria after Liapounoff for infinite triangular matrices" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Sept. 21, 1948, vol. 62, pp. 289-292.

The author considers the system $dx_i/dt = \sum_1^\infty p_{ik}x_k$ with time-dependent coefficients p satisfying the following conditions for all $t > t_1$: (1) $p_{ik} = 0$ if $i < k$; (2) $p_{ii}(t) \leq p_{ii}(t_1) < 0$, $|p_{ii}(t) - p_{ii}(t_1)| \geq |p_{ii}(t_1) - p_{ss}(t_1)| > 0$ if $s \neq i$, $|p_{ii}(t)| > |p_{ss}(t)|$ if $s > i$; (3) $p_{ik}(t) \leq \exp[\alpha_{ik}(t - t_1) - (1 + \beta_{ik}) \log t]$, with certain restrictions on α and β .

For such systems $x_i = 0$ is the limit for $t \rightarrow \infty$ of all motions with sufficiently small initial $|x|$. If, however, p_{ik} for $i > k$ grows faster than $\exp \int_{t_1}^t |p_{ii}| dt$, at least one of the x tends towards infinity if the initial $|x|$ are sufficiently small.

A. W. Wundheiler, USA

159. Ernesto Stagni, "Application of the numerical method to finding the frequency of the free vibration of linear structures (Applicazione del metodo numerico alla ricerca delle frequenze di vibrazione libera di strutture lineari)," *Ann. Mat. pura appl.*, 1947, ser. 4, vol. 26, pp. 85-94.

A numerical procedure is described for the rapid computation of normal modes of elastic structures. The frequency ω of the fundamental mode is computed from Rayleigh's formula $\omega^2 = V(\eta)/T(\eta) = \text{minimum} \dots (1)$, where V , T are proportional to the potential and kinetic energy corresponding to the mode of vibration η . In the case of flexural vibrations of a simple beam this formula requires that $(EJ\eta'')'' = \omega^2 m\eta \dots (2)$, where EJ and m are the stiffness and mass at a given section x of the beam.

The numerical solution of this equation consists of assuming values for $\eta''_{(1)}$ for a small number of stations. Double integration, assuming linear variation between adjacent stations, gives $\eta_{(1)}$. This is substituted in Equation (2) and is integrated again two times to give a second approximation $\eta''_{(2)}$. Another double

integration gives $\eta_{(2)}$. The new values of $\eta''_{(2)}$, $\eta_{(2)}$, are substituted in a sum corresponding to Equation (1), in which a term is added to correct for slope and shear, and bending moment and shear.

Complete solutions for ω are given for several specific cases including the following: rigid clamping of both edges, lateral point load on the plate; simple support along $y = a_1$, rigid clamping along $y = a_2$, uniform lateral pressure; simple support along both edges, hydrostatic pressure.

Walter Ramberg, USA

160. G. A. Nothmann, "Vibration of a cantilever beam with prescribed end motion," *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 327-334.

This paper is a study of the force required on the end of a cantilever beam to maintain a prescribed type of end motion. Four types of end motion are considered: uniform acceleration; uniform acceleration, uniform velocity; uniform acceleration, uniform velocity, uniform deceleration, rest; and sinusoidal pulse, rest. Equations are derived by the use of the Laplace transform for the deflection and shearing force at any point in the beam, and curves are plotted for typical conditions. Attention is called to the fact that these types of motion may be encountered in studies of aircraft wings and fuselages during landing, and of electric-switch blades during opening and closing of the switch. It is interesting to note that the direction of the force may be opposite to the direction of motion under certain conditions.

R. M. Wingren, USA

161. Yves Rocard, "On the auto-oscillation conditions of vibrating systems (Sur les conditions d'auto-oscillation des systèmes vibrants)," *Proc. phys. Soc. Lond.*, Nov. 1948, vol. 61, pp. 393-402.

This is a simple and elementary discussion of familiar facts about self-sustained oscillations and the criteria of stability. The emphasis is on physical exemplification of the basic factors involved. The examples include a bicycle, coupled electric circuits, an airplane wing flutter and an automobile. In the author's Equation (1) it appears that the third term should be $(K/m)(1/l_1 + 1/l_2)^2$, to make the equation dimensionally consistent.

B. S. Cain, USA

162. K. Federhofer, "Computation of the fundamental frequency of a uniformly loaded thin circular plate (Berechnung der Grundschnitzzahl der gleichmässig belasteten dünnen Kreisplatte mit grosser Ausbiegung)," *Öst. Ingen. Arch.*, Sept. 1948, vol. 2, no. 5, pp. 325-331.

In an earlier paper the author considered the deformations of a thin circular plate under uniformly distributed transverse loads if the deflections are of a magnitude comparable to, or larger than the thickness of the plate. On the basis of these previous results the author determines the fundamental frequency of small vibrations of such a plate around its deflected position of equilibrium.

Hans H. Bleich, USA

Wave Motion, Impact

(See also Revs. 242, 272)

163. K. A. Rakhmatoolin and G. S. Shapiro, "On the propagation of plane elastoplastic waves" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, July-Aug. 1948, vol. 12, pp. 369-374.

This paper considers the longitudinal strain-wave propagation in a semi-infinite cylindrical rod subjected to a pulse of load on the end. The pressure is considered to be of sufficient magnitude to cause plastic flow; a piecewise linear stress-strain law and a linear

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work-hardening stress-strain relationship is assumed. Two particular problems are considered: (1) gradual increase and decrease of the load, with the maximum value held for a finite time; (2) instantaneous rise and fall of load.

The equation of motion and the stress-strain relationship for elastoplastic flow yield a hyperbolic pair of linear partial differential equations with velocity and strain as the dependent variables. This pair of equations changes when unloading starts. Methods of a previous paper in the same periodical (1946, vol. 10, pp. 597-604) are applied. The first problem is solved by using the method of characteristics in the plastic and elastic regions. A numerical procedure is outlined.

The second problem involves only wave fronts of stress discontinuity, and the conditions across such wave fronts are obtained from momentum change considerations which determine the development of the wave system. The author assumes that the particle velocity is small compared with the wave velocity, but this restriction would not be necessary if Lagrange coordinates were adopted. The statement of the phenomena occurring is given without proof.

The solution of this type of problem for an arbitrary stress-strain relationship has been given by Bohnenblust [*Nat. defense Res. Council Memo. No. A-47M*]. See also White and Griffis [*J. appl. Mech.*, Dec. 1947, vol. 14, pp. 337-343] for conjectures based on qualitative rather than quantitative considerations.

E. H. Lee, USA

164. W. L. Esmeijer, "On the dynamic behaviour of an elastically supported beam of infinite length, loaded by a concentrated force" (in English), *Appl. sci. Res. Sec. A*, 1948, vol. 1, no. 2, pp. 151-168.

The author investigates the problem of a concentrated impact load on a beam of infinite length supported on an elastic foundation. The intensity of the distributed reaction forces in the foundation is assumed to be proportional at every point to the deflection of the beam at that point, and the foundation material is also assumed to possess a viscous damping resistance. By a suitable change in the variables the fundamental differential equation is written in terms of dimensionless quantities and then analyzed using the Laplace transformation, leading to solutions in terms of tabulated Bessel functions of fractional parameters. Four types of force functions are investigated: square, triangular and two sinusoidal shock waves.

The author reaches a very interesting conclusion in establishing the fact that the maximum bending moment in the beam always occurs within the duration of the impact force, and that its magnitude is inversely proportional to the square root of the impact time for all impacts of the same character and of the same impulse value. The proportionality factor is derived for each of the four types of force functions, and at the end of the paper an outline is given of a method for the analysis of related problems where the impact force is an unknown function of time, as it is in the case of a mass falling on a beam initially at rest.

M. Hetényi, USA

Elasticity Theory

(See also Revs. 172, 207)

165. S. E. Beerman, "On the problem of the elastic equilibrium of an infinite strip" (in Russian), *Notes Acad. Sci. USSR Doklady Akad. Nauk SSSR*, Sept. 11, 1948, vol. 62, pp. 187-190.

To study the stresses in an infinite strip due to normal and shearing forces on its edge and in its plane the author makes use

of the equations of Kolosov from his Russian book written in 1935, *Application of the Complex Variable to Elasticity Theory*. The author develops a general theory and applies it to the case of a strip acted upon by two compressive forces on each side of the strip and in the middle of its length. The results obtained are compared with those found by Filon and Melan. The author also considers the case of a semi-infinite strip.

Witold Wierzbicki, Poland

166. H. A. Elliott, "Three-dimensional stress distributions in hexagonal aeolotropic crystals," *Proc. Camb. phil. Soc.*, Oct. 1948, vol. 44, pp. 522-533.

The conditions for equilibrium in an elastically stressed transversely isotropic medium are formulated by the author. Crystals of closely packed hexagonal form such as those of zinc and magnesium belong to this class of materials. For such materials the elastic behavior is characterized by five elastic moduli.

In the general three-dimensional case, solutions are found in terms of two "harmonic" functions ϕ_1, ϕ_2 which are solutions of the equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} \right) \phi_i + \nu_i \frac{\partial^2 \phi_i}{\partial z^2} = 0 \quad (i = 1, 2)$$

where x and y are taken parallel to the hexagonal planes, z is normal to these planes, and ν_1, ν_2 are roots of a certain quadratic equation. In the case of axially symmetric stress systems, it is shown that the solution may be expressed in terms of a single stress-function Φ .

From these results, the solutions for an isotropic medium may be deduced as a special case. The problems of nuclei of strain in such a hexagonal solid are solved, and the results for zinc and magnesium are compared with those for an isotropic solid.

C. T. Wang, USA

167. A. Philippidis, "A relation between the nonlinear theory of elasticity and the work-hardening theory of Roß-Eichinger-Schmidt (Eine Beziehung zwischen der nichtlinearen Elastizitätstheorie und der Verfestigungstheorie von Roß-Eichinger-Schmidt)," *Z. angew. Math. Mech.*, Apr. 1947, vol. 25-27, pp. 31-32.

The author points out that the stress-strain relations which Roß, Eichinger, and Schmidt have proposed for strain-hardening materials can also be interpreted as nonlinear stress-strain relations for an elastic material. (The author's definition of the term "elastic" is somewhat formal: it does not imply the existence of an elastic potential. However, the result of the present paper remains valid even if a more conventional definition of the term "elastic" is used.)

Courtesy of Mathematical Reviews

W. Prager, USA

168. S. E. Beerman, "Problems on thin-walled members" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Sept. 21, 1948, vol. 62, pp. 305-308.

The stress distribution in thin-walled members is investigated as a problem in plane stress. Neglecting local flexure, the continuity along the various joints of the component thin plates is established by shear stresses exclusively. The paper actually consists of the solution of two specific problems: (1) a thin-walled square member acted upon by purely torsional moments arbitrarily distributed longitudinally, and (2) a thin-walled I-section built-in at one end and loaded along the top of the web by longitudinal external loads, linearly increasing. Derivations are

highly condensed and for nomenclature reference is made to a previous article [*Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Sept. 11, 1948, vol. 62, p. 187; see Rev. 165, this issue APPLIED MECHANICS REVIEWS], which is not available to the reviewer.

It would be interesting to compare the author's results with those obtained from the elementary solution of such problems by Ehlers and Craemer, which has long been in use in Germany in reinforced concrete design [see "Hipped plate construction," *Geo. Winter and M. Pei, J. Amer. concr. Inst.*, vol. 18, no. 5; see Rev. 80, APPLIED MECHANICS REVIEWS, January 1948]. However, omission of nomenclature and details of derivation made such a comparison impossible.

George Winter, USA

169. Sir Richard Southwell, "Relaxation methods applied to engineering problems, XIII. The flexure and extension of perforated elastic plates," *Proc. roy. Soc. Lond. Ser. A*, May 27, 1948, vol. 193, pp. 147-171.

In this paper the author presents an important extension of the range of relaxation methods in dealing with the flexure and extension of multiply connected plates having internal boundaries (holes) along which either stresses or displacements may be specified. The method of approach is exemplified by the solution of the biharmonic equation $D^2 w = Z(x, y)$. In this equation the unknown function w denotes deflections in the flexural problem, and is replaced by Airy's stress-function χ in the analogous extensional problem.

In the case of flexural problems, a solution is obtained by assuming that each internal boundary is subjected to a "clamp" which imposes the specified variations in deflection and slope, and the action of which is subsequently resolved by the well-known "block-displacement" procedure.

In the case of extensional problems a particular difficulty arises from the fact that relaxation methods require a single-valued (acyclic) definition of χ and its first derivatives along the boundaries, in contradiction to orthodox biharmonic analysis which is concerned only with the second derivatives. It is shown that χ or its two first derivatives will entail cyclic values if the resultant moment or the two resultant force components are different from zero along an internal boundary. In order to obtain solutions in such cases, and to avoid the possibility of self-straining which is always present in the analysis of multiply connected bodies, a general method is presented for the elimination of cyclic constants.

This makes it possible to treat analytically a large class of problems which could heretofore be studied only by experimental means. This is strikingly illustrated by a numerical example in which the pattern of lines of equal principal stress difference is calculated for a symmetrical eyebar loaded in axial tension through bolt holes.

M. Hetényi, USA

170. L. Castoldi, "Variational deduction of the equations of dynamics of deformable continua (Deduzione variazionale delle equazioni della dinamica dei continui deformabili)," *Nuovo Cim.*, June 1948, vol. 5, ser. 9, pp. 140-149.

The author uses tensor notation to derive from Hamilton's principle the equations of finite deformation of elastic bodies in invariant form.

A. W. Wundheiler, USA

171. A. Weigand, "The problem of torsion in prismatic members of circular segmental cross section," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1182, Sept. 1948, pp. 1-27 (transl. from *Luftfahrtforsch.*, Feb. 8, 1944, vol. 20).

The problem indicated in the title is treated by the author as one of Saint Venant torsion in terms of a stress function

φ satisfying the equation $\nabla^2 \varphi = -1$. A solution of the equation is adopted in the form of a series of functions, each containing an arbitrary coefficient. The exact boundary condition $\varphi_c = 0$ is replaced by the requirement that the line integral of φ_c^2 along and around the complete boundary be a minimum with respect to variation of the arbitrary coefficients. This leads to a system of simultaneous linear algebraic equations in which the unknowns are the arbitrary coefficients.

The author finds that six terms are sufficient to give results which are close to those of the known solution for the semicircle. The method is then applied to four other segmental angles. The shearing stress at two points on the boundary and the torsional rigidity are computed and compared with tests which were made by a method described previously by the author [*Luftfahrtforsch.*, 1943, vol. 20; also *Nat. adv. Comm. Aero. tech. Memo.*, no. 1179, Sept. 1947].

R. D. Mindlin, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 160, 164, 171, 177, 182)

172. Francisco Garcia Olano and Enrique D. Fliess, "Comments on the calculations of beams of great height, simply supported" (in Spanish), *Cienc. Tecn.*, Nov. 1948, vol. 111, pp. 288-298.

This is a discussion of a recent paper (see Rev. 57, APPLIED MECHANICS REVIEWS, January 1948) on the distribution of stress in short beams [E. Butty, "The simply supported beam of rectangular section and great height," *Cienc. Tecn.*, Oct. 1947, vol. 109, p. 195]. The authors find some errors in Butty's calculations, and after their corrections are made the results agree much better with the results obtained for this problem by Bay using the finite difference method, and by Guzmán and Luisoni using the Galerkin variational method. There are still some differences, however, and the authors suggest a photoelastic test (which does not seem easy to realize). The maximum disagreement (about 30 per cent) between the three solutions is at the bottom of the beam where the maximum tensile stress takes place. The authors think that this disagreement is not important from a technical point of view. (However this would not be the case when materials with little ductility are used, nor when fatigue is involved.)

A. J. Durelli, USA

173. A. E. Johnson, "The stress-strain characteristics of a magnesium-alloy beam," *Airer. Engng.*, Nov. 1948, vol. 20, pp. 330-334.

The work described in this paper was undertaken to investigate the cyclic-loading behavior of a magnesium-alloy beam deformed by a uniform bending moment at room temperature. The rectangular cross section of the beam was 3 in. \times 1 in. The material was alloy DTD289 of the approximate composition: Al, 8 per cent; Zn, 0.5 per cent; Mn, 0.035 per cent; Mg, balance.

Cyclic loading in one direction was applied in increments, while strains were measured with electric strain gages at a number of points on the cross section. The maximum load produced 0.1 per cent permanent set in the extreme fibers on the initial application of load. Auxiliary tensile and compressive cyclic tests were made on specimens taken from low-stressed portions of the beam, and stress-strain relations were determined for the ranges of strains encountered at various distances from the neutral axis in the beam test. These auxiliary data were used to interpret the strains measured in the beam test in terms of bending moment.

The measured strains indicated that plane sections remained plane. There was considerable disagreement between the values of bending moment calculated from the strain readings and the

applied moment. The agreement improved with each cycle of loading until at the eighteenth cycle the agreement was fair. Cyclic moment-strain curves were found which exhibited hysteresis; however, at eighteen cycles the loop apparently reached a stable condition and was reproducible. The stress-strain curves from the auxiliary tests showed similar characteristics.

Marshall Holt, USA

Plates, Disks, Shells, Membranes

(See also Revs. 162, 165, 168, 169, 177, 178, 180, 181)

174. W. R. Leopold, "Centrifugal and thermal stresses in rotating disks," *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 322-326.

The computation of the thermal and centrifugal stresses in a thin rotating disk of variable thickness is discussed under the assumption that the temperature at any point can be approximated by the expression $t_i + kr^n$, where t_i is the temperature at the center of the disk, r is the radius and k and n are constants. The method of solution is similar to that given by Timoshenko in his *Strength of Materials* except that the equation for the displacements of an element of a uniform disk contains an additional term due to the nonuniform temperature. After solving this equation the variable-thickness disk is attacked by replacing it with a series of uniform-thickness disks. The stresses in the actual disk are approximated by using Grammel's graphical construction. A sample computation is given.

J. Paul Walsh, USA

175. H. M. Sengupta, "On the bending of an elliptic plate under certain distribution of load—III" (in English), *Bull. Calcutta math. Soc.*, June 1948, vol. 40, pp. 53-63.

In this paper the author deals with the problem of a thin elliptic plate which is clamped along its boundary and is subjected to a load which is uniformly distributed along an arbitrary portion of the line joining the focal points. A solution is derived by integration from the results of a previous paper by the author [*Bull. Calcutta math. Soc.*, 1948, vol. 40, p. 17] in which the same plate was analyzed under the action of a single concentrated force located between the foci.

Since this original solution had a singularity at the point of application of the force, the principal problem in the present paper is to show that the derived solution remains finite and continuous throughout the entire elliptic area, including the boundary.

M. Hetényi, USA

176. F. László, "Rotating disks in the region of permanent deformation," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1192, Aug. 1948, pp. 1-27 (transl. from *Z. angew. Math. Mech.*, vol. 5, Aug. 1925).

Beginning with the case of a rotating ring, the author shows that as the speed is increased there is first a stable condition in which the strain in the ring is maintained at constant speed. Instability occurs at a speed such that $d\sigma/d\epsilon = 2\sigma/(1 + \epsilon)$, where σ is the true stress and ϵ the unit strain. At this condition the ring continues to stretch uniformly at constant speed by virtue of the added centrifugal force produced by the deformation. However, if the speed is reduced according to a prescribed law, controlled deformation is possible in the unstable region until the condition $d\sigma/d\epsilon = \sigma/(1 + \epsilon)$ is reached, beyond which control is rendered impossible by the initiation of necking.

The case of the rotating disk is then treated in general outline and by analogy to the rotating ring. An equation for instability is derived, but the varying shape of the disk during deformation prevents a simple solution. By making simplifying assumptions

the parallel-sided disk is treated qualitatively. Again there is found a stable region, an unstable region in which deformation can be controlled by reduction in speed, and a necking region which represents the limit of "commercial advantage" of plastic deformation, although the author speculates on the possibility of deformation control in this region also. The importance of the relation between the shape of the stress-strain curve and the strain rate in connection with deformation in the unstable and necking region is discussed.

A typographical error may be noted in the paragraph between Equations (5) and (6). The expression $1 + \epsilon/2$ should read $(1 + \epsilon)/2$. In Fig. 4, for careful tracing of the deformation process, the curve for $T = f(F)$ should pass through zero at $F = F_0$, and the curve for ω should have a maximum exactly at the initiation of instability.

S. S. Manson, USA

Buckling Problems

177. N. J. Hoff and S. E. Mautner, "Bending and buckling of sandwich beams," *J. aero. Sci.*, Dec. 1948, vol. 15, pp. 707-720.

Formulas are given for the deflections of sandwich beams and the buckling loads of sandwich columns. These are derived by means of the energy method, and the results are different from the classical ones for isotropic beams and columns in that the shear energy of the core material is included. Good agreements between theory and experiments were obtained for specimens with balsa core. Discrepancies found for specimens made with expanded cellulose acetate core are attributed to the presence of local weak spots in the core material. Conrad C. Wan, USA

178. F. T. Barwell and J. R. Riddell, "The wrinkling of sandwich struts," *Rep. Memo. aero. Res. Council. Lond.*, no. 2143, June 1946 (issued in 1947), pp. 1-11.

Experimental results on ten sandwich struts composed of steel faces and expanded "formvar" core under compression are reported. The crippling loads for failure of the wrinkling type are compared with a theoretical formula due to H. L. Cox ["Sandwich construction and core materials, Part III. Instability of sandwich struts and beams," *Rep. Memo. aero. Res. Council. Lond.*, no. 2125, Dec. 1945]. The mean experimental value is found to be about 16 per cent higher than Cox's theoretical value.

Conrad C. Wan, USA

179. E. H. Schuette, "Hyperbolic column formulas for magnesium-alloy extrusions," *J. aero. Sci.*, Sept. 1948, vol. 15, pp. 523-529.

This reports flat-end compression tests of 89 Z-section columns of M, FS-1, J-1, O-1, and O-1HTA alloys. After adjustment for end-fixity, it was found that fairly simple curves, different for each alloy, fitted the test results in the elastic range. The curves were significantly below the Euler curves as the stress increased. Horizontal cutoffs at 90 to 100 per cent of the compressive yield stress are recommended. Test data of other investigators are considered and it is concluded that the proposed formulas fit test data better than the Euler formula using the tangent modulus, or formulas of the Rankine and Johnson type.

John E. Goldberg, USA

180. A. Pflüger, "The buckling of anisotropic rectangular plates (Zum Beulproblem der anisotropen Rechteckplatte)," *Ingen.-Arch.*, 1947, vol. 16, no. 2, pp. 111-120.

An analysis is given of the buckling of thin flat rectangular plates, reinforced with stiffeners parallel to the edges and attached

on one side of the plate, under uniform compression and shear forces. Because of the asymmetrical stiffening the author considers extensional, in addition to bending, deformation of the middle surface during buckling. There are then three equations defining the condition of instability, involving the three components u , v and w of the displacement of the middle surface.

The general problem is next particularized to that of a uniform compression force alone. A particular integral of the instability equations is then, in the usual notation, $u = A \cos(m\pi x/a) \sin(n\pi y/b)$, $v = B \sin(m\pi x/a) \cos(n\pi y/b)$, $w = C \sin(m\pi x/a) \sin(n\pi y/b)$. The corresponding expression for the buckling stress coefficient is derived. The results are presented in a design chart for the practical case of stiffening parallel to the direction of applied load.

H. G. Hopkins, England

181. D. Y. Panov and V. I. Feodosov, "Stability of shallow shells under large deflections" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), July-Aug. 1948, vol. 12, pp. 389-406.

The author follows the standard procedure to derive a system of 14 equations in 14 unknowns characterizing the equilibrium of thin shallow asymmetric shells under large deflections. Previous formulations of similar problems were concerned with flat plates and with special cases of shells of revolution subjected to loading which preserves spherical symmetry. The resulting equations are too lengthy to be quoted here. They appear in the paper as five equations numbered (2.1), three equations (3.3), (3.6), (3.8), and six equations (4.3). These equations lead to the equation for deflection w in the anticipated form

$$-p - D\nabla^4 w + T_x \frac{\partial^2(w_0 + w)}{\partial x^2} + 2Q \frac{\partial^2(w_0 + w)}{\partial y \partial x^2} + T_y \frac{\partial^2(w_0 + w)}{\partial y^2} = 0,$$

where $w = w_0(x, y)$ is the equation of the middle surface before deformation, T_x and T_y are the compressive forces and Q the shearing force, p the lateral load, and D the flexural rigidity.

As an illustration of the solution of this equation the author considers a uniformly loaded circular plate so freely clamped along the contour as to permit free displacements in the radial and tangential directions (that is, $T = Q = 0$ on the contour) and to preclude angular displacements. This mode of clamping admits the formation of folds which destroy the circular symmetry. This problem, discussed on pp. 396-406, is solved by Galerkin's method. As one would expect, the computations leading to the determination of the amplitude of folds and for the instability criteria are very lengthy. I. S. Sokolnikoff, USA

Structures

(See also Rev. 178)

182. Arne Hillerborg, "A study of dynamic influences of moving loads on girders" (in English with French and German summaries), *Publ. int. Ass. Bridge Struct. Engng.*, 1948, Third Congress, prelim. publ., pp. 661-667.

The problem of a concentrated load moving at a constant speed along a uniform girder is studied. When damping is neglected the dynamic increment is completely defined by the dimensionless parameters $\alpha = (\text{velocity of load}) / (2 \times \text{natural frequency of girder} \times \text{length of girder})$, $\nu = (\text{mass of load}) / (\text{mass of girder})$.

Previous results of Inglis [*A Mathematical Treatise on Vibrations in Railway Bridges*, Cambridge, 1934] are improved, and an approximate simple formula is obtained for the free vibration

component. Dynamic increments for the deflection and the maximum moment are computed for a range of the above two parameters. Good agreements between the theoretical and experimental results are found.

Conrad C. Wan, USA

183. A. M. Freudenthal, "Inelastic behaviour and safety of structures" (in English with French and German summaries), *Publ. int. Ass. Bridge Struct. Engng.*, 1948, Third Congress, prelim. publ., pp. 643-650.

This paper consists of comments on the following topics:

(A) The benefit to an indeterminate structure of yielding in the metal and consequent redistribution of stress. (B) Different aspects of wholly or partly irreversible energy absorption in the metal, such as "relaxation by spontaneous heat energy fluctuations" or creep; "elastically restrained inelastic deformation" or residual strain (the above benefits being due to action of this sort); "crystal fragmentation" or strain-hardening. (C) The meaning of "structural damage" and its identification with "fragmentation of the internal structure." (D) Safety of a structure, as indicated by its deformation.

No detailed analysis or application is given or suggested.

W. P. Roop, USA

184. C. D. Crosthwaite, "Analysis of the long span suspension bridge" (in English with German and French summaries), *Publ. int. Ass. Bridge Struct. Engng.*, 1948, Third Congress, prelim. publ., pp. 435-450.

The author considers the analysis of long suspension bridges. Following the procedure of Timoshenko and Priestner of substituting a trigonometric series for the exponential form of equation, the author obtains a solution which takes account of variable hanger pull. Southwell's relaxation procedure is then applied to the trigonometric series to obtain a solution which eliminates the usual simplifying assumptions that the hanger pull is constant along the cable, the hangers are inextensible, the cable displacements in each span are vertical, the cable slides over rigid towers and the stiffening truss is of uniform rigidity.

The method is applied to a suspension bridge of 33,000-ft span. An evaluation of the results confirms the fact that of these simplifying assumptions only the effects of nonuniform truss rigidity and hanger extensions are important in long-span bridges.

Karl Arnstein, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 163, 173, 176, 183, 202)

185. F. R. N. Nabarro, "Mechanical effects of carbon in iron," *Rep. Conf. Strength Solids, Univ. Bristol*, July 1947 (publ. in 1948), Physical Society, London, pp. 38-45.

Cottrell has shown that foreign atoms tend to diffuse to regions where their strain energy is lowest, and that the stress around a dislocation may be relieved by a migration of solute atoms in its neighborhood. Once the diffusion of foreign atoms has established this pattern the force required to move the dislocation is increased. This mechanism and the unusually large diffusion coefficient of carbon in α iron are cited to explain the existence of a yield point, quench aging, strain aging, delayed yield and blue brittleness of iron containing a small amount of carbon.

The yield point, with its drop in load, is considered to be characterized by the large force required to tear a dislocation from its position as "locked-in" by the carbon atoms, and by the continued flow at a smaller stress after the dislocation is clear of the carbon field. The explanation of quench aging assumes hardening to be

caused by the formation of cementite after quenching from an annealing temperature below which austenite is formed. Strain aging is caused by the anchoring of dislocations, by the diffusion of carbon from compressed to dilated regions in their neighborhood. Delayed yield pictures the external stress to move a dislocation slightly, which causes a diffusion of carbon atoms from their original positions; after a time the anchoring of the dislocations is weakened and the dislocation suddenly breaks away from its carbon atmosphere. At higher temperatures the mobile carbon atoms are able to follow the dislocation and exert a viscous drag and a larger stress must be applied before the dislocation breaks away. This explains the phenomenon of blue brittleness.

Experimental curves showing the relation between the aging rate and the temperature are in fair agreement with numerical values involving the diffusion coefficient of carbon.

Irwin Vigness, USA

186. W. Prager, "Discontinuous solutions in the theory of plasticity," *Courant Anniv. Vol.*, Interscience Publishers, New York, 1948, pp. 289-300.

The author extends the theory of plastic flow to those cases where a discontinuous surface is present across certain stress components which may be discontinuous. This development is preceded by an excellent review of the standard mathematical theory of plasticity.

Clarence Zener, USA

187. G. Gurevich, "On the law of deformation of solid and fluid bodies" (in Russian), *J. tech. Phys. (Zh. tekhn. Fiz.)*, Dec. 1947, vol. 17, pp. 1491-1502.

This paper is designed to prove that the well-known relaxation equation of Maxwell, presented in the transformed form

$$\frac{d\sigma}{d\epsilon} = K_0 - \sigma / \left(\tau \frac{d\epsilon}{dt} \right) \dots \dots \dots (1)$$

(where ϵ is relative strain, σ is stress, t is time and τ is a coefficient proportional to the "relaxation time") expresses, in first approximation, the general relation involved in the fundamental laws of straining of amorphous or polycrystalline bodies.

The author develops from this, as a particular case, the following expression for strain velocity v under constant load:

$$v = \frac{d\epsilon}{dt} = \frac{\epsilon_{\text{elast}}}{\tau} = \frac{E_0 \cdot \epsilon_{\text{elast}}}{E_0 \cdot \tau} = \frac{\sigma_{\text{const}}}{\eta} \dots \dots \dots (7)$$

The above equation is used by the author instead of the following one used by many other investigators:

$$v = \frac{\epsilon_0}{\tau} = \frac{\text{const}}{\eta}$$

Incidentally the author emphasizes that "the very widespread denial that Equation (7) has any physical meaning is paradoxical."

M. T. Huber, Poland

188. F. C. Frank, "On slip bands as a consequence of the dynamic behaviour of dislocations," *Rep. Conf. Strength Solids, Univ. Bristol*, July 1947 (published in 1948), Physical Society, London, pp. 46-51.

This paper gives a theoretical, mainly qualitative discussion of the dynamics of dislocation movement in crystalline solids. The author points out that the ratio of the kinetic energy to the strain energy of a moving dislocation increases toward a number of the order of unity as the velocity approaches that of a transverse sound wave. In the absence of large dissipatory influences it is

unlikely that a fast-moving dislocation will intersect a free surface without causing a reflected dislocation. Dislocations reflected from free surfaces are always of such a sign as to continue the shearing motion characteristic of the incident dislocations. From similar considerations it appears that fast-moving dislocations meeting in a glide plane pass over or reflect so as to continue the active shearing process. Additional assumptions to explain why glide continues when once started appear unnecessary.

The more pertinent inquiry as to what causes stopping of a once-started glide in a single crystal leads to a plausible account of how glide planes "seize-up" with fixed screw dislocations. The distance between seized-up glide planes is estimated by the author at about 100 atomic spacings. This, as well as other details of the author's dislocation dynamics, is consistent with Heidenreich's observations of slip-band fine structure.

The author suggests that single crystals, if deformed when immersed in a fluid of equal or greater density, will have a changed slip-band fine structure. Suppressing of free-surface reflections should permit increases in the amount of glide between glide lamellae prior to seize-up.

George R. Irwin, USA

189. E. N. da C. Andrade, "The creep of metals," *Rep. Conf. Strength Solids, Univ. Bristol*, July 1947 (published in 1948), Physical Society, London, pp. 20-26.

In this paper the author first presents a description of proposed creep experiments under a condition of homogeneous shear. Next he attempts an explanation of the phenomenon of accelerated flow in the last stage of constant-load creep experiments. This phenomenon is found to be due primarily to the increase in stress, but in certain cases the author considers it to be due also to structural changes taking place in the metal while the flow is in progress. A description of these structural changes is given.

Finally the author presents a discussion of the physical distinction between what he calls β -flow and k -flow. These two forms of flow are defined by the author by means of his formula $l = l_0(1 + \beta t^{1/3})e^{kt}$, which gives the gage length l of the specimen as a function of the time t at constant stress. The exponential term represents a constant rate of flow per unit length, which the author calls k -flow. The term $\beta t^{1/3}$ represents a rapidly decreasing rate of flow which he calls β -flow.

Aris Phillips, USA

Failure, Mechanics of Solid State

(See Revs. 185, 191, 263)

Design Factors, Meaning of Material Tests

(See Revs. 183, 191)

Material Test Techniques

190. Kenneth A. Baab and Hobart M. Kraner, "Sonic method for determining Young's modulus of elasticity," *J. Amer. ceram. Soc.*, Nov. 1948, vol. 31, pp. 318-320.

The authors suggest the sonic method of determining modulus of elasticity as a nondestructive control test for refractories. The modulus of elasticity of brick was computed from the natural frequency of a free-free bar by vibrating it with an electrodynamic vibration motor. The modulus of several samples was plotted versus the firing temperature, and good sensitivity to differences in the firing temperature was found. Other tests such as for strength, density and porosity did not give equal sensitivity.

R. O. Fehr, USA

Mechanical Properties of Specific Materials

(See also Revs. 173, 188, 190)

191. Ernst Raub, "The influence of hard chromium plating on the fatigue strength of aluminum alloys (Der Einfluss der Hartverchromung auf die Dauerfestigkeit von Aluminiumlegierungen)," *Metallforsch.*, Apr. 1947, vol. 2 (*Z. Metallk.*, vol. 38), pp. 121-126.

The electroplating of other metals upon aluminum alloys has in recent years received commercial acceptance. For instance heavy chromium deposits are used to reduce friction, increase wear resistance and increase resistance to alkalines. But the fatigue strength of the plated material is generally substantially reduced. Research workers early found that the chromium plating itself is not important, and that the reason for the reduction of fatigue strength is to be found in the treatment of the aluminum surface to put it in good condition for conventional plating methods.

In the paper reviewed the results from very comprehensive tests are reported. The materials tested included Al-Mg alloys and Al-Mg-Zn alloys. They were tested in rotating bending and in vibratory reversed bending. The effect of some of the etching methods used to produce a satisfactory surface for electroplating were studied, as well as the effect of the thickness of the chromium deposit. From the tabulated results it is easy to see that the reduction of fatigue strength is due to the etching treatment, and that increasing the thickness of the chromium deposit has a slightly favorable effect.

Ragnar Nilson, Sweden

192. R. G. McElwee, "Section size, hardness, composition relationship in gray cast iron," *Soc. auto. Engrs. J.*, Nov. 1948, vol. 56, pp. 33-35 (full paper in preprint form, 7 pp.).

The paper is introduced by a general discussion of the effects of the chief alloying elements in cast iron: carbon, silicon, manganese, sulphur and phosphorus. The author sets out to justify a higher carbon level in terms of a lower tendency to microporosity (due to a shorter freezing range), a lower percentage of scrap in the risers (since the necessity for feeding is less) and a higher damping capacity. Assuming that the effect of additional silicon on the structure and hardness is about one third that of the same amount of additional carbon, a high tensile strength can be maintained by reducing the silicon content as the carbon is increased, to keep the "equivalent carbon" constant.

Although the Brinell hardness of high-carbon irons is low, due to a greater graphite content, the hardness of the pearlitic matrix is not affected, and the irons have satisfactory wear resistance. The formation of white iron at the surface of thin sections is discussed, and it is suggested that this may be prevented by inoculation treatment. Diagrams show the tensile strength and hardness as functions of the equivalent carbon content and section thickness.

B. W. Mott, England

193. V. C. F. Holm and A. I. Krynsky, "Observations on the control of grain size in magnesium casting alloys," *J. Res. Nat. Bur. Stands.*, Sept. 1947, vol. 39, pp. 265-270.

The methods investigated for the control of grain size in magnesium casting alloys were: (1) stirring of carbonaceous solids into the molten alloy; (2) bubbling of carbon monoxide through the melt; (3) melting in a silicon carbide crucible; and (4) treatment with lump magnesite. In the last method, the magnesite was placed deep in the molten metal where it dissociated thermally, generating a carbonaceous gas which bubbled through the melt. The grain size and tensile properties were determined for specimens from each melt, and these were compared with data

from melts which had been superheated as well as those which had received no special treatment.

The results indicate that magnesium alloys which have been properly treated with carbonaceous materials, possess grain size and tensile properties equivalent to those obtained with superheated metal. The magnesite method seems to be a simple, inexpensive and convenient means of developing a fine-grained structure in magnesium casting alloys.

H. M. Schnadt, Luxemburg

Mechanics of Forming and Cutting

194. R. E. McKee, R. S. Moore, and O. W. Boston, "An evaluation of cylindrical-grinding performance," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1948, vol. 70, pp. 893-901.

This is the third in a series of papers by the authors, and presents results of the effect of such variables as wheel grain, grade, velocity, and type of material on the grinding operation. The authors make use of a relationship they name the "grinding rating" for comparison. This rating is the ratio of the volume of metal removed to the volume of wheel consumed, called the volume ratio, divided by the product of the horsepower per unit volume per minute and the surface finish. The materials studied include SAE 1020, 1045 and 52100 steels at BHN of 131, 260, and 653 respectively, and two cast irons, one BHN 187, the other BHN 229. Only one grinding compound, an emulsion type, was used.

The authors find that the best grinding results were obtained at peripheral speeds of 6500 fpm; that specific horsepower goes down with decreasing grain size, and slightly up with increasing hardness of the wheel; that a grinding rating above 0.154 would indicate favorable practice; increased table feed or depth of cut results in poorer finishes; and that aluminous-oxide wheels can be used satisfactorily on cast irons. In the discussion and author's closure it is pointed out that individual wheels are not of consistent properties from face to face along the radius, presenting the most difficult control problem in tests of this type.

Morton B. Millenson, USA

Hydraulics; Cavitation; Transport

(See also Rev. 208)

195. L. Prandtl, "The nature of surface tension (Zum Wesen der Oberflächenspannung)," *Ann. Phys.*, 1947, vol. 1, nos. 1-3, pp. 59-64.

The author establishes the fact that the laws of statics show surface tension to be a real tension (compare a thin film). This tension S may be calculated from the laws of molecular attraction. The author concentrates on the case of attraction equal to er (r = distance) for $r < R$, and zero for $r > R$. This yields $S = (\pi c/48)\rho^2 R^6$ (ρ = density). If the attraction is $\varphi(r)$ for $r < R$, $S = (\pi\rho^2/48) \int_0^R R^6 \varphi(R) dR$.

Hans Thygesen Kristensen, Denmark

196. W. T. Bottomley, "Erosion due to incipient cavitation," *Proc. Instn. mech. Engrs.*, 1948, vol. 158, no. 3, pp. 297-316.

The classic work on the complete life history of a cavitation bubble, comprising its growth, collapse and final rebound, was recently published by Knapp and Hollander ["Laboratory investigation of the mechanism of cavitation," *Trans. Amer. Soc. mech. Engrs.*, July 1948, pp. 419-435]. In the present paper, the subject matter of which was primarily based on the data supplied

by van Iterson, the author advances certain ideas about the formation and collapse of bubbles causing cavitation erosion, and compares them with the existing theory of Cook and Parsons propounded in 1919 and by Rayleigh in 1920.

He suggests that the bubbles which cause erosion are ordinary air bubbles filled with air at roughly atmospheric pressure, and that their collapse is due to surface tension only and is not a result of the sudden change in water pressure. These suggestions are not beyond the range considered by others like Beeching and Nunachi; nevertheless the discussion presented by the author portrays his own theory based on his experimental framework rather than a further development of the general theory of cavitation.

The experiments also show that vapor bubbles which collapse in deaerated water are in a state of thermal equilibrium. The energy which produces cavitation erosion is the free-surface energy liberated by the collapse of the air bubbles, while the final collapse velocity is the velocity of sound in water, with the magnitude of impact pressure of the order of 120 tons per sq in. or more. The actual effects of these ideas on the interpretation of model tests in cavitation tunnels are described.

There is an interesting discussion in which some of the authors' ideas are refuted and shown to be invalid. This reviewer also feels that the true conception of the mechanism of cavitation erosion can only be had after these experimental results are fully evaluated so as to fit in the general framework. The paper, though very controversial in its nature, forms a lively springboard for further discussion and research, and a notable contribution to the widely scattered literature on cavitation erosion.

S. K. Ghaswala, India

197. C. H. Johansson, "Theoretical investigation of the effect of capillary suction on transfer of moisture in hygroscopic materials" (in English), *Trans. roy. Inst. Technol. Stockh.*, 1948, no. 20, pp. 3-15.

The author develops an elementary theory for transfer of moisture through a single capillary tube, considering moisture transfer to be effected both by diffusion of vapor and by capillary suction of liquid. This theory is then extended to plates veined with capillaries of varying diameter, with a brief mention of possible application to a porous body composed of closely packed spheres.

The theory could have been better developed on the basis of a free-energy gradient as discussed by Edlefsen and Anderson [*Hilgardia*, Feb. 1943, vol. 15, no. 2]. The author's use of a distribution function to treat the systems of variable diameters will be of interest to researchers in the field of petroleum reservoir mechanics.

J. A. Putnam, USA

198. A. M. Binnie and G. A. Hookings, "Laboratory experiments on whirlpools," *Proc. roy. Soc. Lond. Ser. A*, Sept. 2, 1948, vol. 194, pp. 398-415.

Tests are described on the discharge of water through small models of submerged trumpet-shaped circular weirs with radial, tangential, and combined radial and tangential approach of the flow in the reservoir. A deep and a shallow bellmouth were used (entrance diameter 6.25 in., discharge pipe ID 1.279 in., depth of conical sections 8.96 and 2.96 in. respectively) and compared with a plain pipe. In all cases the radial supply without tangential component gave the highest discharge.

The head-discharge curve shows two branches: one for the flow controlled by the weir action on top of the bellmouth with an air core occupying the center of the discharge pipe; and a suddenly upturning branch indicating the rapid rise in head when the pipe

flows full and the carrying capacity of the pipe controls the flow. With tangential and mixed radial and tangential flow the full-flow point is reached at much reduced capacity, as compared with radial approach (most reduced for the plain pipe, less for the deep, and least for the shallow bellmouth). Surging was observed on the shallow bellmouth at the change from flow with air core to the full-flow condition.

An analytical expression is derived for the discharge with air core and calculated and observed data are compared. The variation of the constant c giving the distribution of swirl in the circular tank (according to $c = vr$) was determined experimentally by measuring the time of rotation of lead-laden match sticks floating vertically in the whirling water. Otmar E. Teichmann, USA

199. J. C. Schönfeld, "Resistance and inertia of the flow of liquids in a tube or open canal" (in English), *Appl. sci. Res. Ser. A*, 1948, vol. 1, no. 3, pp. 169-197.

This study, dealing with unsteady flow in tubes or canals with constant cross sections of different shapes, is intended to demonstrate the dependence of the potential drop on resistance and inertia. By means of asymptotic approximations, respectively for quickly or slowly varying motions, it shows that the potential drop depends in the first case practically on inertia only, but in the second one on inertia and resistance. In this case the resistance is greater than for steady motion, due to a different velocity distribution. The study is conducted for different situations of unsteady flow, turbulent as well as laminar. In each case the expressions are given for a function R of the resistance and for two functions H and L of inertial forces, which the author calls high-frequency or low-frequency inertances. The latter is always greater than the former, and a theoretical value of the ratio of the two functions is given for the laminar motion.

Duilio Citrini, Italy

200. Francesco Ramponi, "On the evaluation of oscillations in a surge tank (Sul calcolo delle oscillazioni in un pozzo piezometrico)," *Energia Elett.*, Sept. 1948, vol. 25, pp. 499-503.

If the height of the surge tank is not small compared with the gallery length, the usual calculation omitting the inertia of the liquid contained in the tank is not satisfactory. Taking into account this inertia, oscillations of greater amplitude and period are obtained. However, many experiments conducted on different models at the Laboratory of Hydraulics in Padua have shown that the formulas furnished in this case by the theory are well verified with regard to the period, but not the amplitude, which in effect becomes smaller.

The author explains this fact by means of an initial loss of kinetic energy due to elastic causes (water hammer). With the aid of an auxiliary hypothesis, he derives a formula well in accord with the experimental results.

Duilio Citrini, Italy

Incompressible Flow: Laminar; Viscous

(See also Revs. 198, 235, 241, 249, 254, 273, 276, 280, 281, 283)

201. Clifford Truesdell, "A formula for the vortex vector of a viscous elastic fluid (Une formule pour le vecteur tourbillon d'un fluide visqueux élastique)," *C. R. Acad. Sci. Paris*, Oct. 27, 1948, vol. 227, pp. 821-823.

It is found that vorticity in a viscous elastic fluid may be considered to arise through three different mechanisms. The first is that of any nonconservative force field present, modified by a fictitious force field due to viscosity and the existing vorticity: $\vec{f} - (\mu/\rho)\Delta \times \vec{\zeta}$ in vector notation. The second is that of a

baroclinic density field (Bjerknes theorem), with the pressure modified by a fictitious pressure due to viscosity and expansion, $p + (2\mu + \lambda)D \log \rho / Dt$. The third is that of nonuniformity of the viscosity field, the effect being dependent upon the deformation tensor as well as upon the expansion and viscosity gradient.

The results are presented in mixed Lagrangian and Eulerian co-ordinates using tensor notation. Wallace D. Hayes, USA

202. Hans Fromm, "Laminar flow of Newtonian and Maxwellian liquids (Laminare Strömung Newtonscher und Maxwellscher Flüssigkeiten)," *Z. angew. Math. Mech.*, Aug.-Sept. 1947, vol. 25/27, pp. 146-150.

The author points out that the time rate of the stress tensor which appears in the stress-strain law of Maxwell is the material rate which can be represented as the sum of the local and convective rates and a third rate due to the rotation of the volume element. The resulting theory is applied to the laminar flows between parallel plates and in a tube of circular cross section.

Courtesy of Mathematical Reviews

W. Prager, USA

203. M. N. Tipei, "Corrections in a wind tunnel with a closed throat bounded by circular arcs (Corrections dans un tunnel aérodynamique à veine fermée, limitée par des arcs circulaires)," *C. R. Acad. Sci. Roum.*, 1946-1947, vol. 8, pp. 184-188.

The interference potential for a single vortex line (uniformly loaded wing) was given by Kondo (1935) for an air stream constrained by a pair of circular arcs, using bipolar co-ordinates. Kondo's formula is now applied to a vortex sheet with general Γ -distribution given in the usual form of a sine series, as in Glauert's theory. The integral representing the additional mean velocity v_m at the location of the wing is, in the case of elliptic Γ -distribution, approximately evaluated by a series development of the integrand, and the obtained accuracy is estimated by comparison with Prandtl's formula as specialized for a circular tunnel. Formulas are given that permit one to evaluate v_m in the same approximate way for a general Γ -distribution.

G. Kuerti, USA

204. Albert May and Jean C. Woodhull, "Drag coefficients of steel spheres entering water vertically," *J. appl. Phys.*, Dec. 1948, vol. 19, pp. 1109-1121.

Steel spheres having diameters from $1/4$ to $1 1/2$ in. were shot vertically into water at speeds ranging from 25 to 208 fps. Decelerations were measured by means of a high-speed movie camera. From these data the authors have obtained drag coefficients during the cavity phase of the sphere's flight. The basic data so obtained are believed to be more extensive and accurate than other published results.

The reviewer disagrees with the methods that were used to analyze the results. The drag is influenced by such factors as viscosity, water density, velocity, diameter, sphere density, depth, air pressure, vapor pressure and gravity. Consequently the drag coefficient should be analyzed as a function of such variables as Reynolds number, Froude's number, cavitation parameter, ratio of depth to diameter, and ratio of water to sphere density.

The authors varied only the velocity and the diameter of the spheres and choose to ascribe the resulting effects solely to Reynolds number and Froude's number, although the unanalyzed data on effect of atmospheric pressure indicate that a cavitation parameter is of at least equal importance. They subtract the force of hydrostatic buoyancy and gravity. Believing that the remaining drag should follow a V^2 law, they plot $1/V$ versus t for each run to obtain the drag. In this process they arbitrarily

reduce the corrections for buoyancy and gravity by 56 per cent to obtain straight lines. The inconsistency of this is seen by noting that their final data show that the drag is proportional to $V^{2.13}$ which should give a straight line for $V^{-1.13}$ versus t . Summarizing, the authors are incorrect in attributing the drag coefficient variations with velocity and length solely to Reynolds and Froude's number effects, and the adjustments that have been made of the data are questionable and may have introduced errors comparable with the small variations found.

Francis H. Clauser, USA

205. G. W. Platzman, "Some remarks on the measurement of curvature and vorticity," *J. Met.*, Apr. 1947, vol. 4, pp. 58-62.

Since many of the recent theoretical meteorological studies involve ζ , the component of the vorticity normal to the earth's surface, it is necessary to measure ζ experimentally in synoptic studies of these problems. The author points out that for any two-dimensional fluid motion on a surface, the vorticity component normal to the surface can be written in the intrinsic form

$$\zeta = S + VG$$

where S is the velocity shear normal to the streamline, V is the fluid velocity and G is the geodesic curvature of the streamline. Explicit formulas are given for computing G on a spherical earth from data represented on a sphere and on the commonly used polar stereographic projection. Some applications of these results to the computation of constant vorticity trajectories are also presented.

H. J. Stewart, USA

206. M. J. Lighthill, "Notes on the deflection of jets by insertion of curved surfaces, and on the design of bends in wind tunnels," *Rep. Memo. aero. Res. Council. Lond.*, no. 2105, Sept. 1945 (issued in 1947), pp. 1-7.

The commonly observed phenomenon that a jet of water can be deflected by partial insertion of a curved surface (convex toward the axis of the jet) is analyzed by means of the classical potential theory. A similar treatment is applied to the two-dimensional analysis of the case in which walls replace the sides of the jet. The velocity is constant along the walls except in a small limited region where deleterious effects of adverse velocity gradients can be checked by means of moving walls—such as a rotating cylinder—or of suction slots. The contour of the bend is obtained through conformal transformation of properly chosen patterns in the hodograph plane. Of particular interest is the design of a 180-deg bend with all adverse gradient avoided by use of one suction slot. Such a bend would eliminate the need for cascades of turning vanes in wind tunnels and reduce the number of bends from four to two.

Joseph V. Foa, USA

207. D. I. Sherman, "On Prandtl's equation in the finite-wing theory" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekhn. Nauk)*, May 1948, no. 5, pp. 595-600.

This is a highly condensed account of an application of meromorphic function theory to the exact solution of Prandtl's (so-called finite-wing) integro-differential equation in the special case where the chord has the form of a rational function of x times $(a^2 - x^2)^{1/2}$ (where x represents the distance from the plane of symmetry and a the semispan). The complex function $\phi(z) = (2\pi i)^{-1} \int_{-a}^{+a} \Gamma(t) dt / (t - z)$, which has the property $\phi(z)|_{y=+0} - \phi(z)|_{y=-0} = \Gamma(x)$ (where Γ is the unknown circulation and z an auxiliary complex variable, $x + iy$, in a plane similar to that of Trefftz) is determined, in principle, by considering the

singular points of the various functions occurring in the transformed Prandtl equation. The method is said to be capable of solving equations similar to Prandtl's, but with complex coefficients, arising in certain plane problems in elasticity. The reviewer recalls, for instance, those considered by N. I. Muskhelishvili [*Math. Ann.*, 1932, vol. 107, pp. 282-312].

M. V. Morkovin, USA

208. A. Graham Foster, "Pore size and pore distribution," *Disc. Faraday Soc.*, 1948, no. 3, pp. 41-51.

The values of capillary radii in porous solids may be determined either by using the capillary theory or by estimating the surface area of the solid. In the former case, if condensation occurs, the Kelvin equation furnishes the pore radius directly. In the latter case the condition $S/V = 2/r$, where S is the internal surface area and V volume of a cylindrical solid, also permits approximate estimating of the value of the capillary radius r . Besides these methods, if sorption ceases after formation of a monolayer, it may be assumed that the pore radius is of the order of one or two molecule diameters. Curves and numerical data for silica, alumina and ferric oxide gels are given. D. P. Krynine, USA

Compressible Flow, Gas Dynamics

(See also Revs. 236, 244, 245, 256, 265)

209. A. Robinson, "Rotary derivatives of a delta wing at supersonic speeds," *J. roy. aero. Soc.*, Nov. 1948, vol. 52, pp. 735-752.

This paper constitutes an extension of the work of Puckett and Stewart and others on the aerodynamic characteristics of delta wings at supersonic speeds. The methods and results of these authors are used to develop expressions for the roll damping due to roll, and pitch damping due to pitch. It is shown that by the linear theory, while the chordwise center of pressure due to incidence is at $2/3$ of the root chord from the apex, the center of pressure due to pitching around the apex is at $3/4$ of the root chord from the apex. The general solutions for these stability derivatives are obtained and plotted versus Mach number for various values of semiapex angle. Both the "quasi-subsonic" (leading edges inside the Mach cone) and the "definitely supersonic" (leading edges ahead of the Mach cone) cases are considered.

E. Arthur Bonney, USA

210. Chi-Teh Wang, "Variational method in the theory of compressible fluid," *J. aero. Sci.*, Nov. 1948, vol. 15, pp. 675-685.

For steady irrotational motion of a compressible fluid it is shown, following the work of Bateman, that the first variation of the integral over the whole volume of fluid, $I = \int_V p d\tau$, where p is the pressure and $d\tau$ is an element of volume, is zero. However, when the fluid domain is infinite, (1) the integral becomes infinite, and (2) Bateman's principle requires modification in the form of a correction term. It is shown that the first difficulty may often be overcome by expressing the velocity potential as the sum of two parts, ϕ_1 and ϕ_2 , of which the first corresponds to incompressible flow, and does not enter into the variational principle. The variational integral then becomes $\int_V [p(\phi) - p(\phi_1)] d\tau$ and, under certain circumstances, this is finite even when extended to infinite volume. It is shown in the particular case of a circular cylinder how the second difficulty may be circumvented by a partially *a priori* analysis which yields the proper correction term.

The application of these ideas to the problem of compressible flow past a circular cylinder at Mach number 0.4 is developed in detail. The velocity potential ϕ_1 of the incompressible flow is

well known, and the added potential ϕ_2 is the limit of a sequence of functions all of which satisfy the boundary conditions but with undetermined constants. Following the Rayleigh-Ritz method, the variational procedure then gives a series of nonlinear algebraic simultaneous equations for the undetermined constants. The labor of computation can be much reduced by employing the theorem of residues and Crout's method for solving the simultaneous equations. Using six constants, the results are found to compare well with the Rayleigh-Janzen method, as worked out by Imai, and with the electrical analog of Taylor and Sharman. The effect of using fewer constants is also shown.

In treating the flow past an arbitrary airfoil, it is difficult to prescribe the boundary conditions in simple analytical form. This at first appears to be a stumbling block in the Rayleigh-Ritz method, for it is then difficult to select functions which satisfy the boundary conditions. However, it is shown that the boundary conditions are preserved in a conformal mapping. Thus, assuming that the function conformally mapping a circle into an arbitrary airfoil is known, the determination of the compressible flow past the airfoil may be found by the Rayleigh-Ritz method.

The further development of these methods, especially for flows with circulation, and for mixed subsonic and supersonic flows, will be awaited with interest.

Ascher H. Shapiro, USA

211. R. E. Grey and H. D. Wilsted, "Performance of conical jet nozzles in terms of flow and velocity coefficients," *Nat. adv. Comm. Aero. tech. Note*, no. 1757, Nov. 1948, pp. 1-32.

This NACA technical note presents flow coefficients and velocity coefficients determined by the authors from tests on 15 nozzles, for conical jet nozzles in the range of cone half angles of 5 deg to 90 deg and of outlet-to-inlet diameter ratios of 0.50 to 0.91. The inlet diameter was in all cases 5 in. The flow coefficients were found to depend on cone angle, outlet-to-inlet diameter ratio, and pressure ratio. The velocity coefficients were essentially constant below the critical pressure ratio. Above the critical pressure ratio they decreased slightly, but were in all cases independent of cone angle and diameter ratio. Thus the variation in air flow and thrust in nozzles having the same air flow and thrust at some particular design point was proportional to their flow coefficients. The results are given in the form of about a dozen curves. These results are intended for application to the design of jet-propulsion nozzles.

Charles Concordia, USA

212. Chin Shih Ken, "Remarks on the approximate integration of the equations of the continuous movement of a compressible fluid (Remarques sur l'intégration approchée des équations du mouvement continu d'un fluide compressible)," *C. R. Acad. Sci. Paris*, Oct. 27, 1947, vol. 225, pp. 718-722.

The author investigates a two-dimensional fluid flow under the assumptions of the following equations of state (1) $p = p_0 - A/\rho$, $q^2 = A/\rho^2 + B$, and (2) $p = p_0 + A \int \rho^2 d\rho / (\rho^2 - B^2)$, $q^2 = A/(\rho^2 - B^2)$, in order to determine which of these fictitious equations of state approximates the adiabatic equations of state more closely. The author concludes that (1) is a better approximation for subsonic speeds while (2) is better for supersonic speeds.

Abe Gelbart, USA

213. J. Nicolas, "Application of series expansions to supersonic irrotational flows (Application des développements limités aux écoulements supersoniques et irrotationnels)," *Rech. aéro. Paris*, July 1948, no. 4, pp. 47-49.

The author considers axial or plane symmetric supersonic potential flow for a given velocity distribution along the axis of symmetry. Neglecting v^2 and $\partial v^2 / \partial x$ (v is transverse velocity),

the author determines the power-series expansions of the velocities in the neighborhood of the axis in terms of their axial values. An approximate graphical construction is hence derived.

A. W. Wundheiler, USA

214. J. S. Isenberg, "The method of characteristics in compressible flow," *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-1173 A and B-ND, Dec. 1947, pp. 1-219 and 1-51 (plus 5 charts).

Existing procedures using the method of characteristics for the numerical solution of problems in steady supersonic gas flow are reviewed, summarized, and in some cases extended. An inviscid perfect gas with constant specific heats is considered in both two-dimensional and axially symmetric flow. The emphasis is placed upon practical application to calculations. The method of characteristics is developed for the general case of rotational flow with variable rest-enthalpy, and each special case is treated in full.

Numerical, graphical, and combined integration procedures are developed, using both lattice-point and mesh methods, and using the following combinations of dependent variables in the compatibility equations (characteristic equations in hodograph or related variables): (1) flow velocity, flow angle, and entropy; (2) flow velocity, flow angle, and shock-wave angle; (3) static pressure and flow angle; (4) flow velocity components in the two Mach directions; (5) certain functions, λ and μ , of the flow velocity and flow angle, and the total pressure ratio (on a given streamline) across a shock wave.

Various initial conditions are discussed, for flow in channels and around bodies. The theory of shock waves is developed, and procedures are given for use in conjunction with the previously developed integration procedures. An extensive bibliography is included.

Part IA of the report contains charts and tables for use in the various procedures with air.

W. G. Cornell, USA

215. Jack N. Nielsen and Edward W. Perkins, "Charts for the conical part of the downwash field of swept wings at supersonic speeds," *Nat. adv. Comm. Aero. tech. Note*, Dec. 1948, no. 1780, pp. 1-32.

A sweptback wing may be considered to be a triangle with the tips cut off and a V-shaped notch cut to form a trailing edge. The induced-flow field is the sum of a conical part contributed by the triangle in the absence of any cuts (infinite chord), and non-conical parts originating at the tips and trailing edges. In the present paper analytical expressions are derived for the downwash throughout the entire induced-flow field of lifting triangles of infinite chord at supersonic speeds, with either supersonic or subsonic leading edges. Downwash charts are included from which the downwash field may be determined for any practical combination of leading-edge sweep and flight Mach number. The conical part of the downwash field of a swept wing is obtained from the charts, and the nonconical contributions of the tips and trailing edges must be determined by other means such as the method of lift cancellation employed by Lagerstrom and others.

Herbert S. Ribner, USA

216. J. K. L. MacDonald, "On some problems involving linearization of aero-thermodynamical equations," *Courant Anniv. Vol.*, Interscience Publishers, New York, 1948, pp. 241-251.

Illustrations are given to show advantages in the use of Lagrangian co-ordinates in connection with linearization of problems in gas dynamics. A one-dimensional acoustic wave equation in

Lagrangian variables is shown to involve neglect of fewer terms than the more usual Eulerian analog. Major space is given to mathematical treatment of equations representing the propagation of a somewhat idealized flame along a tube. Lagrangian co-ordinates are again used; a solution of a linear approximation is given, and both unsuccessful and successful iteration procedures for higher approximations are discussed. To test the accuracy of the linear approximation an alternative numerical solution based on difference equations is given. The agreement is stated to be within one per cent generally.

Philip Rudnick, USA

217. A. Robinson and A. D. Young, "Note on the application of the linearized theory for compressible flow to transonic speeds," *Coll. Aero. Cranfield Rep.*, no. 2, Jan. 1947, pp. 1-8.

It is pointed out that according to the linearized theory the lift-curve slope of a wing with finite span but arbitrary shape remains finite and continuous through the transonic range. The authors neglect to emphasize the very powerful limitation of the linearized theory in the transonic range to angles of attack (or thickness ratio) satisfying the similitude condition that the quantity $\alpha/[M^2-1]^{1/2}$ be small.

Wallace D. Hayes, USA

218. A. Kahane and Lester Lees, "Unsteady one-dimensional flows with heat addition or entropy gradients," *J. aero. Sci.*, Nov. 1948, vol. 15, pp. 665-670.

This paper deals with the problem of unsteady one-dimensional flow of a gas with heat addition and variation of entropy. The general one-dimensional equations of unsteady flow are set up and the procedure for their numerical integration by the method of characteristics is formulated.

The transient flows, which arise when heat is added to a section (as in a combustion chamber) of an initially isentropic flow in a tube, are calculated. The physical significance of the calculated results is discussed. In particular, the process of transition from an unsteady state to the final state of steady flow is discussed. This clarifies certain obscure points in a theory where the flow is assumed to be steady to begin with (see also Revs. 332 and 1668, APPLIED MECHANICS REVIEWS, February and November 1948).

C. C. Lin, USA

219. H. W. Liepmann and A. E. Puckett, "Introduction to aerodynamics of a compressible fluid," John Wiley & Sons, Inc., New York, 1948. Cloth, 9.2×5.8 in., 262 pp., 122 figs.

As stated in the preface, the book is the basis for a fifty-hour graduate course at the California Institute of Technology, but represents no attempt to produce a standard classroom textbook. The reviewer feels that the book stresses the facts rather than their explanation and organization, and addresses itself to the engineer rather than to the physicist. (For instance, the interpretation of the speed of sound on page 26 is based on the consideration of steady flow in a cylindrical pipe; but such a flow is piecewise uniform at arbitrary velocity, with stationary shocks. On page 50, Mach lines in steady flow are discussed as if the flow were a transient.) The method of presentation is a mixture of derivation and description, without a clear-cut separation of the two approaches.

Details sufficient to equip the reader for an independent attack on problems are given only in the scalar theory of one-dimensional channel flow, in the small perturbation method for two-dimensional flow and in the theory of Prandtl-Meyer flow. Other topics are treated by means of brief surveys of the problems involved.

Steady one-dimensional flow in channels, including shock waves, is treated first. A deviation from systematic organization

is created by the discussion of two-dimensional (oblique) shocks at this point. Also two-dimensional reflection is treated here in order to complete the discussion of nozzle and wind-tunnel phenomena. A very instructive discussion of wind-tunnel design and measurements (optical methods) concludes the chapter on one-dimensional flow.

Two-dimensional flow theory starts with the derivation of the equations and a discussion of basic flow concepts. Linearization by the perturbation method is taken up first. Flow past a wavy wall is discussed at length. The Prandtl-Glauert rule is derived, and applied to basic supersonic thin airfoil theory. Two approximate methods (including the Rayleigh-Janzen one) are sketched. The hodograph method and the associated approximations (Chaplygin, von Kármán-Tsien and the von Kármán 1941 method) are carefully presented.

Turning now to exact solutions, the Prandtl-Meyer flow is treated analytically, and the Ringleb solution is presented in some detail. The method of characteristics follows with descriptive discussions of certain wave interaction problems. The last two chapters give a brief review of viscosity phenomena (boundary layers, laminar stability and turbulence), of high-speed airfoil theory and of transonic phenomena. An oblique-shock chart and a characteristics diagram are attached to the book.

A. W. Wundheiler, USA

220. C. Ferrari, "Supersonic flow fields about bodies of revolution," *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-29, 1948, pp. 1-25 (transl. from *Aerotecnica*, 1937, vol. 17, no. 6, pp. 507-518).

This is a translation of an *Aerotecnica* article of 1937, one of three written about the same time by the author on high-speed flow about bodies of revolution.

In this paper the linearized theory is treated with particular reference to the flow about bodies of revolution at angles of attack. In addition, the nonlinearized method of characteristics for the supersonic flow about bodies of revolution at zero incidence is presented. Historically this was the first appearance of these two developments in gas-dynamical theory.

Wallace D. Hayes, USA

221. Chieh-Chien Chang, "The transient reaction of an airfoil due to change in angle of attack at supersonic speed," *J. aero. Sci.*, Nov. 1948, vol. 15, pp. 635-655.

The problem of the reaction of a plate in a supersonic flow of velocity U , on a change of the angle of attack, is solved by the linearized theory. The velocity potential at a point $P(x, y, z)$ due to a source is given by Garriek and Rubinow's formula.

Superposing these sources, the velocity potential for a sheet of line sources, parallel to the η axis, extending from $\xi = 0$ to $\xi = \xi_1$, turns out to be Possio's solution,

$$\phi(x, y, t) = \frac{-1}{\pi \sqrt{M^2 - 1}} \int_0^{\xi_1} \int_{\tau_1}^{\tau_2} \frac{v(\xi, +0, t - \tau)}{\sqrt{(\tau - \tau_1)(\tau_2 - \tau)}} d\tau d\xi, \\ y > 0,$$

where v is the vertical velocity, immediately above the airfoil (downwash), τ_1 and τ_2 are the times needed for a cylindrical wave to travel from the line source at $Q(\xi, +0)$ to $P(x, y)$. For $y < 0$ an analogous formula holds.

The downwash distribution is calculated from the assumed angle of attack α , which is zero for $t < 0$, varies linearly from $t = 0$ up to $t = t_1$ and for $t > t_1$ remains equal to α_1 . The time derivative $\dot{\alpha}$, as well as α itself, being expressed by Dirichlet integrals, are introduced in the formula for the velocity potential. The necessary integrations are performed, using integral representa-

tions for Bessel functions. The velocity potential is discontinuous along certain lines in the x, t plane owing to the discontinuities in the prescribed downwash at $x = 0$ and at $t = 0$ and $t = t_1$. This gives rise to three regions for the parameter $v_0 = x/ut$, separated by the values $1 - 1/M$ and $1 + 1/M$ and likewise for the parameter $v_1 = x/[u(t - t_1)]$, which regions partly overlap the first triplet of regions.

The value of ϕ in each of these regions is calculated, as well as the values of C_L and C_M and the wave drag. Numerical results for several values of the parameters M , α_1 and t_1 are given. Finally the calculations are generalized for the case of an arbitrary variation of the angle of attack with the time, the time derivative in this case being approximated by a step function.

It is shown that a finite time $C/(U - c)$ (C = chord length, c = sound velocity) after all change in angle of attack has ceased, the transient reaction, which is a rather intricate function, is completely damped out, the other contributions to C_L (or C_M) consisting of a term, proportional to α_1 (Ackeret's formula) and a term proportional to $\dot{\alpha}$, which in the stationary state is zero.

R. Timman, Holland

222. R. C. Roberts, "The method of characteristics in compressible flow. Part II. Unsteady flow," *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-1173D-ND, Dec. 1947, pp. 1-90.

The method of characteristics is applied to the solution of problems in the unsteady one-dimensional flow of a perfect gas. External forces, friction and heat conduction are neglected. Spherically and cylindrically symmetric flows are considered, as well as flow in channels of slowly varying cross section. Emphasis is placed on the application of the theory to computation. Graphical methods are developed for isentropic flow. More accurate numerical methods are developed for nonisentropic flow. Boundary conditions are discussed for initial-value problems (velocity, pressure, and entropy prescribed along a noncharacteristic curve in the x, t plane), for flow between fixed or moving cross sections, and for discharge under constant pressure.

Shock waves are treated separately. The general theory is developed. Approximate graphical and numerical methods are developed for weak shock waves. An exact method is developed for strong shock waves in constant-area channels. For strong shock waves in other configurations (so that the shock-wave velocity varies with position), a qualitative discussion is given and reference is made to existing literature. Examples are discussed for flows with and without shock waves.

A theory is developed for detonation waves (with constant propagation velocity) in which heat is liberated chemically in the shock front. Analytical solutions from the theory of sound waves are discussed. Reference is made to the literature for analytical solutions in special cases of finite amplitude waves.

W. G. Cornell, USA

223. P. A. Lagerstrom, "Linearized supersonic theory of conical wings," *Nat. adv. Comm. Aero. tech. Note*, no. 1685, Aug. 1948, pp. 1-161.

In conical supersonic flows the velocity vector is constant along any radial line from the origin, or the vertex. The problem can be reduced to two variables by considering the intersections of the radial lines with a fixed plane perpendicular to the undisturbed flow. For points within the "Mach circle" made by the intersection of the Mach cone from the vertex and the representative plane, the differential equation is elliptic; outside of the Mach circle, it is hyperbolic.

If the disturbances are small, the equation can be linearized; then outside of the Mach circle, the solution can be obtained easily

by a modification of the simple two-dimensional solution of Ackeret. Continuity determines then the value of the velocity on the Mach circle, and the boundary condition on the wing the value of the velocity on the horizontal diameter of the Mach circle which is the trace of the wing. The problem is thus reduced to a linear partial equation of elliptic type in the Mach circle, with properly specified boundary conditions. Moreover, the equation can be reduced to the Laplace equation by a simple change in the scale of distances from the origin of the plane of Mach circle. Thus the powerful methods of the theory of functions of complex variable can be applied. This is one important contribution of the present paper.

The other elements are: The application of the Lorentz transformation, or the oblique transformation, and of the principle of superposition of solutions. An aim of this paper is to lay a firm foundation for these methods by a thorough discussion and by various examples. These include flat symmetric wings, flat lifting wings, wings of constant lift distribution and finally a mixed type of lifting wing. Details of numerical solutions for practical engineering calculations must be obtained from the papers listed at the end of the work.

Although none of the essential elements presented originated with the author, as they were first introduced by A. Busemann, R. T. Jones, and others, the author achieved such clarity and precision that his work can be said to be an excellent example of the work of an applied mathematician, having both beauty and utility.

H. S. Tsien, USA

224. Bruce L. Hicks, Donald J. Montgomery, and Robert H. Wasserman, "On the one-dimensional theory of steady compressible fluid flow in ducts with friction and heat addition," *J. appl. Phys.*, Oct. 1947, vol. 18, pp. 891-903.

The present paper is devoted to a study of a steady nonadiabatic flow of a compressible fluid (with friction) on the basis of the one-dimensional approximation. The authors derive in the usual way the differential equations, and using the classical theorems draw some conclusions about the existence and some quantitative properties of the flow. In particular the authors study the changes of the flow if the conditions slightly change. When the local Mach number equals one, a special condition must be satisfied in order that a solution without a shock exists. Finally, choking phenomena are discussed.

The equations considered in the paper are nonlinear; in the opinion of the reviewer the mathematical theorems mentioned in the paper do not constitute a completely sufficient basis for the conclusions drawn. (See also Review 1668, APPLIED MECHANICS REVIEWS, November, 1948.)

Stefan Bergman, USA

225. Frank S. Malvestuto, Jr., and Kenneth Margolis, "Theoretical stability derivatives of thin sweptback wings tapered to a point with sweptback or sweptforward trailing edges for a limited range of supersonic speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1761, Jan. 1949, pp. 1-30.

As the authors' summary states, the stability derivatives valid for a limited range of supersonic speeds are presented for a series of sweptback wings tapered to a point with sweptback or sweptforward trailing edges. These wings were derived by modifying the trailing edge of a basic triangular wing so that it coincided with lines drawn from the wing tips to the wing axis of symmetry. The stability derivatives were formulated by using the pressure distributions previously obtained for the basic triangular wing for angle of attack, sideslip, pitching, rolling and yawing. Explicit expressions are given for the stability derivatives with respect to principal body axes, and conversion formulas are provided for the transformation to stability axes. The results are

limited to Mach numbers for which the wing is contained within the Mach cones springing from the vertex and from the trailing edge of the center section of the wing. M. J. Thompson, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 238, 255)

226. A. A. Townsend, "Experimental evidence for the theory of local isotropy," *Proc. Camb. phil. Soc.*, Oct. 1948, vol. 44, pp. 560-565.

In this work the Kolmogoroff hypothesis of locally isotropic turbulence [G. K. Batchelor, *Proc. Camb. phil. Soc.*, 1947, vol. 43, p. 533] has received strong experimental support for the particular case of the "isotropic" turbulence downstream from a square-mesh biplane grid at fairly high mesh Reynolds numbers.

The verifications presented are (1) constancy of the skewness factor $S(r)$ of $(u_2 - u_1)$ (the instantaneous velocity difference between two points a distance r apart, with r parallel to u_1 and u_2) over the (narrow) range of r for which the double correlation function $1 - g(r)$ follows Kolmogoroff's $2/3$ -power prediction; (2) invariance of skewness and flattening factor of the probability density of $(\partial u / \partial x)$ over sizable ranges of mesh Reynolds number and dimensionless downstream distance.

Related recent papers are: G. K. Batchelor and A. A. Townsend [*Proc. roy. Soc. Lond. Ser. A*, 1948, vol. 193, p. 539] and A. A. Townsend [*Austral. J. Sci. Res.*, 1948, vol. 1, no. 2].

Stanley Corrsin, USA

227. F. N. Frenkiel, "The decay of isotropic turbulence," *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 311-321.

The author proposes theoretical equations for the decay of isotropic turbulence for the case in which the Reynolds number of the turbulence is so large that the correlation curves can be regarded as preserving their shape during decay, and for the case in which the Reynolds number is so small that the triple correlations can be neglected. These cases correspond roughly to neglecting the viscous and inertial terms respectively in the general equations for the correlation coefficients.

In the case of large Reynolds number use is made of Loytsian-ski's assumption that $r^5 R(r)$ approaches zero as r approaches infinity, which leads to the constancy with time during decay of $u'^2 L^*$, where u'^2 is the mean square turbulent velocity fluctuation and $L^* = [\int_0^\infty s^4 R(s, t) ds]^{1/5}$. This assumption combined with the assumption of self-preserving correlation curves leads to a decay law without arbitrary constants as follows:

$$\overline{u'^2} = \overline{u_0'^2} [7\nu/\lambda_0^2(t - t_0) + 1]^{-10/7}$$

In the case of small Reynolds number, the decay law depends on the initial shape of the correlation curve. Decay laws are worked out for several empirical shapes.

The available experimental data are discussed at some length, but the comparisons are made difficult by the absence of data on the quantity L^* . When the transverse scale $\int_0^\infty R(y) dy$ is used in place of L^* , the agreement is poor, and the discrepancies are attributed to imperfect isotropy in the experiments.

Hugh L. Dryden, USA

228. M. Ray, "Flow of a liquid from a reservoir over a plane-boundary-layer theory," *Phil. Mag.*, May 1948, vol. 39, pp. 409-412.

The boundary-layer equations of Prandtl including the time variation are considered. The problem is reduced to that of a

nonlinear ordinary differential equation by introducing the variables $\psi = \nu^{1/2} f(\eta) x t^{-1/2}$, where $\eta = y(\nu t)^{-1/2}$, obtained through the usual similarity considerations. The differential equation is

$$f'''(\eta) + \{f(\eta) + \frac{1}{2}\eta f''(\eta) - (f'(\eta))^2 + f'(\eta)\} = 0.$$

The appropriate solution is found by considering a series of positive powers of η , beginning with the quadratic term. The author finds a satisfactory approximation by restricting himself to four terms of the series, the coefficients of which are easily obtained. The author interprets the solution physically in terms of the nonsteady flow of a viscous liquid from a reservoir over a semi-infinite flat plate.

Courtesy of *Mathematical Reviews*

F. E. Marble, USA

229. Hans Peter Schmitz, "On the theory of turbulent exchange (Zur Theorie der Austauschströme)," *Z. Met.*, Mar. 1948, vol. 2, pp. 71-77.

The main point of the paper is a new definition of an equivalent displacement in turbulent exchange of a property of amount $e(x, y, z)$ per unit volume. If δe stands for the instantaneous deviation of e from the mean value $\bar{e}(x, y, z)$, the equivalent displacement is defined as a vector $\xi_i (i = 1, 2, 3)$, normal to the equiscalar surface $\bar{e} = \text{const}$, its absolute value being the distance along this normal between the equiscalar surfaces corresponding to $\bar{e} + \delta e$ and \bar{e} . The turbulent transport of e is then given by $E_i(e) = -\bar{\xi}_i \bar{e} \partial \bar{e} / \partial x_i$ where $\bar{\xi}_i$ are the components of eddy velocity. Resolving the eddy velocity component $\bar{\xi}_i$ into the component $^*\bar{\xi}_i$ normal to the surface $\bar{e} = \text{const}$ and the component $^0\bar{\xi}_i$ normal to $^*\bar{\xi}_i$ and $\bar{\xi}_j$, the exchange tensor $\bar{\xi}_i \bar{\xi}_j$ can be resolved into a symmetric and antisymmetric part, and consequently the component of the turbulent transport expressed in the form

$$E_i = -\mu \frac{\partial \bar{e}}{\partial x_i} - \bar{\psi}_j \frac{\partial \bar{e}}{\partial x_k} + \bar{\psi}_k \frac{\partial \bar{e}}{\partial x_j} \quad (i \neq j \neq k),$$

with $\mu(e) = \bar{\xi}_n \bar{\xi}_n$ a scalar corresponding to a diffusion coefficient and $\bar{\psi}$ a vector with components $\bar{\psi}_k = -^0\bar{\xi}_i \bar{\xi}_j$, ($i \neq j \neq k$, negative sign for an odd order in i, j, k , otherwise positive).

Introducing this new definition of the equivalent displacement, the author reduces the number of six independent components of the exchange tensor in Ertel's tensor theory of turbulence [*Ann. Hydrograph.*, 1937, vol. 65, pp. 193-205] to only four independent components. Using the above expression for the turbulent transport, the author derives the general equation for turbulent exchange of e , as well as of $e' -$ the corresponding amount of the property for a unit mass. He neglects the turbulent variation of density in the expressions for the exchange tensor, since its relative variation is 10^{-2} times smaller than the relative variation in velocity components.

Z. Sekera, USA

230. G. K. Batchelor and A. A. Townsend, "Decay of turbulence in the final period," *Proc. roy. Soc. Lond. Ser. A*, Nov. 9, 1948, vol. 194, pp. 527-543.

Supplementing earlier work on the decay of turbulence in the early stage (near a grid) this work describes theoretical and experimental results in the later stage (further downstream) where the inertia terms in the equations of motion can be neglected. The results of earlier work [Batchelor, *Quart. appl. Math.*, July 1948, vol. 6] are described briefly. These predict that in the final period $\bar{u}^2 \propto t^{5/2}$ and $f(r, t) = \exp(-r^2/8\nu t)$ where \bar{u}^2 is the mean square value of the velocity, t is the time, $f(r, t)$ is the correlation between the velocity components in the r direction at a distance r . Since the neglect of the inertia terms makes the velocity components independent, a new approach, similar to that used by

Reissner and utilizing integrals of the equation of motion without the inertia terms, yields further results applicable in the final stage. It is shown that homogeneous turbulence tends to an asymptotic statistical state which is independent of the initial conditions, except perhaps for a multiplicative constant. These results do not depend on isotropy.

Important experimental results confirming some of the theoretical results obtained by means of the hot-wire anemometer are presented. The dissipation length parameter λ , \bar{u}^2/U , and $f(r)$ are measured as functions of x and r for three values of the mesh Reynolds number $R_M = UM/\nu$, where M is the mesh size of the grid used to produce the turbulence. For $R_M = 650$ the "final period" in which the above results are valid appears to prevail for $x/M > 400$. For $R_M = 950$ and 1360 the results are not as conclusive, but they indicate that the final period is reached at about $x/M = 800$. Very good agreement is shown with the $5/2$ -power law and with the exponential law for $f(r, t)$ above. In agreement with an earlier investigation, in the final period $\lambda^2 = 4\nu(x - x_0)/\nu$. The theoretically derived condition that the inertia terms in the equations of motion may be neglected is $R_\lambda = \sqrt{\bar{u}^2 \lambda / \nu} \ll 15$. The experiments show that a necessary condition for the beginning of the final period is $R_\lambda < 5$. It appears therefore, that within the range of these experiments the results given for this final period are indeed in the region in which the inertia terms can be neglected.

A. M. Kuethe, USA

231. H. Ertel and S. Rombakis, "On the theory of the virtual friction coefficient (Zur Theorie des virtuellen Reibungskoeffizienten)," *Z. Met.*, Dec. 1947, vol. 1, pp. 459-461.

This treatment of turbulent transport of momentum, turbulent friction and dissipation is not essentially different from the old phenomenological theories of turbulence because the fluctuation intensity is assumed to be proportional to the gradients of mean motion.

The authors put this proportionality into the following form: Let the absolute magnitude of the turbulent deformation tensor be proportional to the absolute magnitude of the deformation tensor of the mean motion,

$$|D_{ik}| = \alpha |\bar{D}_{ik}|$$

$$\text{where } D_{ik} = \frac{1}{2} \left(\frac{\partial V_i}{\partial x_k} + \frac{\partial V_k}{\partial x_i} \right)$$

and the bar means time average.

In this manner the "turbulent viscosity" becomes α^2 times the molecular viscosity. They regard as experimental verification of the "theory" the fact that by guessing the order of magnitude of α they get the right order of magnitude for the temperature fluctuations of the atmosphere. Leslie S. G. Kovászny, USA

232. Leslie S. G. Kovászny, "Spectrum of locally isotropic turbulence," *J. aero. Sci.*, Dec. 1948, vol. 15, pp. 745-753.

This paper suggests an expression, different from any given previously, for the energy spectrum of isotropic turbulence in the stationary or equilibrium range of wave numbers, first discussed by A. N. Kolmogorov. The new assumption made by the author is that the inertial transfer of energy across any point of the spectrum is a function of the local wave number n and energy density $E(n)$ only, and on dimensional grounds must thus equal $Kn^{5/2} E^{3/2}$, where K is an absolute constant. This leads to a spectrum which varies as $n^{-5/3}$ in the purely inertial range and goes to zero at a certain higher wave number. Fair agreement is found with measurements of the energy spectrum (by Simmons) and of the correlation function (by Corrsin).

The reviewer finds difficulty in justifying the author's new as-

sumption: the transfer of energy across any wave number is essentially the integrated effect of conditions over the neighboring octaves so that the possibility of writing it as a function of the local conditions seems to exist only when there is a simple similarity of the spectrum shape over this range, for instance, when the spectrum has a power-law shape. Also, the author appears not to have appreciated the difference between the one-dimensional and three-dimensional spectrum functions. In the comparison with experiment he has assumed they have the same form but this is in general only true when they both have a power-law form. It is certainly untrue in the range of viscous effects in which the author's results are novel.

G. K. Batchelor, England

233. C. C. Lin, "Note on the law of decay of isotropic turbulence," *Proc. nat. Acad. Sci. Wash.*, Nov. 1948, vol. 34, pp. 540-543.

The Kolmogoroff forms for the double and triple correlation functions (f and h) and the von Kármán-Howarth equation are used to deduce a new law for the decay of isotropic turbulence. The author implicitly assumes that, after substitution of the Kolmogoroff forms into the latter equation, all terms in the resulting equation are of the same order of magnitude. This assumption and some dimensional analysis lead immediately to the deduction of the functional relations between the rate of decay, the characteristic velocity, and the characteristic length which were postulated by Kolmogoroff. They also lead directly to a new law for the rate of decay of the intensity of turbulence. This law appears to agree better with existing experimental evidence than previously suggested decay laws.

The result admits of an interesting interpretation: the concept of a self-preserving correlation function was proposed by von Kármán and Howarth and was to apply to the late (low Reynolds number) stage of turbulent decay. If one interprets this self-preservation as applying only to the small-scale (low wave-length) fluctuations, and if the concept is applied (via Kolmogoroff's correlation-function forms) at the early stages of decay, the decay law of this paper results. Thus the law is a consequence of the concept that the correlation functions are locally self-preserving at both high and low Reynolds numbers.

George Carrier, USA

234. P. Y. Chou, "On velocity correlations and the equations of turbulent vorticity fluctuation" (in English), *Sci. Rep. Nat. Tsing Hua Univ. Ser. A*, Apr. 1948, vol. 5, pp. 52-70.

The author starts from the equations of turbulent vorticity fluctuation which may be obtained by taking the curl of the equations of velocity fluctuation, and which consequently do not contain the pressure fluctuation. From the equations of vorticity fluctuation the author obtained equations which are satisfied by the double correlation functions between the vorticity fluctuation components: we shall denote these equations by (E).

Besides (E) the author considers also the equation of energy transport and makes the three following assumptions: (1) The double and triple velocity correlations between two neighboring points P and P' may be developed into power series of ξ'/λ where ξ' are the components of the vector PP' and λ Taylor's scale of microturbulence; the coefficients of these series are supposed to be linear functions of the Reynolds stress. (2) The variations of the correlation functions due to a rigid translation are negligible compared with the variations of these functions during a relative displacement between the points P and P' . (3) The average energy flux carried by convection through a unit surface perpendicular to a determined direction is directly proportional to the gradient in that direction of the rate at which work is done by

the Reynolds stress in changing the volume and shape of an element of volume, and inversely proportional to the square of the decay of turbulent energy.

According to the results given in an earlier paper [*Quart. appl. Math.*, 1945, vol. 3, pp. 38-51], the author shows that, with these assumptions, the considered equations can be uniquely expressed in terms of the Reynolds stress, of λ and of the velocity of the mean motion.

Finally, besides the mean motion there are seven unknown functions, namely the six components of Reynolds stress and the Taylor's scale of microturbulence. We also have seven equations: the six equations (E) and the equation of energy transport. These seven equations considered together with the equations of the motion and the equation of continuity for the mean flow define all the unknown values.

The author applies the theory to the following problems: (1) The decay of energy and vorticity in isotropic turbulence; the results are verified by the experiments of Batchelor and Townsend. (2) The pressure flow within a channel; with a proper choice of two experimental constants, the theory leads to the logarithmic law of velocity distribution which is known to agree well with observation. For a large portion of the channel and for all three of the mean squares of velocity fluctuation one obtains a linear dependence of the distance from the center of the channel, which is in qualitative agreement with the existing data.

Ratip Berker, Turkey

235. H. Görtler, "Formation of boundary layers on cylinders accelerated from rest (Grenzschichtentstehung und Zylindern bei Anfahrt aus der Ruhe)," *Arch. Math.*, 1948, vol. 1, no. 2, pp. 138-147.

The author develops the Blasius method for the calculation of boundary-layer growth on cylinders which start from rest with velocities of the general form $U(t) = Ct^n$ ($C = \text{const}$, $n \geq 0$, an integer). For the case of an elliptic cylinder he computes the first approximation to the position of the separation point, and gives an estimate of the time in which separation begins.

J. A. Lewis, USA

236. D. Meksyn, "Integration of the boundary-layer equations for a plane in a compressible fluid," *Proc. roy. Soc. Lond. Ser. A*, Dec. 7, 1948, vol. 195, pp. 180-188.

An analytical method (described in a previous paper by the author) is used to integrate the differential equations developed by Emmons and Brainerd for the flow over a flat plate.

The results obtained using the analytical method are shown to compare closely with those previously obtained by Emmons and Brainerd using the differential analyzer. C. A. Meyer, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 207, 209, 215, 221, 225, 252)

237. William M. Kauffman, Allan Smith, Charles J. Liddel, Jr., and George E. Cooper, "Flight tests of an apparatus for varying dihedral effect in flight," *Nat. adv. Comm. Aero. tech. Note*, no. 1788, Dec. 1948, pp. 1-35.

This report covers the development and preliminary tests of an apparatus varying the effective dihedral of an airplane in flight. The device enables the rate of change of rolling moment with angle of sideslip to be varied by means of a servomechanism which actuates the ailerons (and hence provides rolling moment) in response to angle of sideslip. Sideslip angle, the external intelligence, is sensed by conventional means and fed into the control unit. Only a relatively small portion of the total aileron travel is

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required for the device, the remainder being available for manual use. The device is also required to maintain the appropriate stick force and position gradients with sideslip, since the pilot judges the stick-fixed and stick-free dihedral effect by means of these gradients.

In actual test the device functioned satisfactorily and enabled the effective dihedral to be varied over a range of 14.9 deg to -2.7 deg for the stick-fixed condition. During dynamic maneuvers, the pilot could detect lag effects but they were not large enough to vitiate the usefulness of the device.

Many important uses can be found for this apparatus which readily simulates what the pilot may expect from physical changes of the dihedral angle. Its usefulness in connection with the establishment of specifications for manual flying characteristics of aircraft, etc., may be readily seen.

In general, the device represents a special case of artificial stability and control by means of which the various derivatives of the airplane may be modified through the application of servo- and servointelligence systems.

W. F. Milliken, Jr., USA

238. Gerald M. McCormack and Woodrow L. Cook, "A study of stall phenomena on a 45-deg swept-forward wing," *Nat. adv. Comm. Aero. tech. Note*, no. 1797, Jan. 1949, pp. 1-32.

The lift, drag and pitching-moment characteristics were correlated with the pressure distribution, boundary-layer measurements, and tuft studies on a 45-deg swept-forward wing. It was found that at an angle of attack of about 10 deg ($C_L = 0.5$ to 0.7), turbulent separation occurs over the inboard sections, causing an increase in drag, a rearward shift of the aerodynamic center, but no loss in lift. At angles of attack above 16 deg ($C_L = 0.7$ to 1.04), leading-edge separation caused very large increases in drag, a decreased lift-curve slope, and an extremely large forward shift of aerodynamic center.

To improve the characteristics of swept-forward wings for flight purposes, it is proposed that effort should first be directed toward delaying leading-edge separation.

Louis Landweber, USA

239. Paul E. Purser and John P. Campbell, "Experimental verification of a simplified vee-tail theory and analysis of available data on complete models with vee tails," *Nat. adv. Comm. Aero. Rep.*, no. 823, 1945 (issued in 1948), pp. 1-21.

This paper presents a simplified theory for the calculation of the stability and control parameters of the "vee" tail. The longitudinal and directional characteristics are considered independently. The derivation of the formulas and their modification for design purposes is given in detail.

Results of wind-tunnel tests of two isolated vee tails with variable dihedral angle and of two complete airplane models provide a check of the theory; the agreement appears to be fairly good within the limits mentioned in the report.

Finally the advantages and disadvantages of the vee tail are discussed.

J. Buhrman, Holland

240. Harry J. Goett and Noel K. Delany, "Effect of tilt of the propeller axis on the longitudinal-stability characteristics of single-engine airplanes," *Nat. adv. Comm. Aero. Rep.*, no. 774, 1944 (issued in 1947), pp. 1-28.

Tests of a model of a typical single-engine fighter airplane indicate that a considerable reduction of the destabilizing effect of power may be obtained by a downward tilt of the propeller axis. For a prototype with gross weight of approximately 15,000 lb and operating at 2100 hp a favorable shift of the neutral point amount-

ing to five per cent of the mean aerodynamic chord was measured on the model, increasing to 10 per cent at 3450 hp; maneuver stick forces were reduced materially.

It is shown that existing theory will predict results with sufficient accuracy for preliminary design purposes if the down-wash in the slip stream is considered in computing the tail-pitching moment.

John E. Goldberg, USA

241. Lange and Wacke, "Test report on three- and six-component measurements on a series of tapered wings of small aspect ratio," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1176, May 1948, pp. 1-73 (transl. from *Dtsch. Luftfahrtforsch. Untersuch. Mitt.*, no. 1023/5, Sept. 27, 1943).

This report gives the results of three- and six-component wind-tunnel tests of triangular wings of aspect ratios 3, 2, 4/3 and 1. The tests were made with models whose actual span varied from 1.5 meters to 0.866 meters. The airfoil section was the NACA 0012. The tuft studies presented show the flow pattern near to the wing as a function of angle of attack. The indications from extensively tabulated results are that the lower limit of applicability of Prandtl's airfoil theory is given by the aspect ratio 3.

The only wing with a stable moment break at the stall was of aspect ratio 3. With increasing lift coefficient, however, the wings of aspect ratio 2 or less showed an aft movement of the neutral point, but at the stall the moment break was unstable.

Conrad A. Lau, USA

242. D. Williams and R. P. N. Jones, "Dynamic loads in aeroplanes under given impulsive loads with particular reference to landing and gust loads on a large flying boat," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2221, Sept. 1945 (issued in 1948), pp. 1-31.

A straightforward method for calculating the dynamic loading of aircraft under landing or gust loads is described. The method is applied to a hypothetical large flying boat and it is found that landing loads for such configurations are not likely to be more critical than gust loads.

John W. Miles, USA

Aeroelasticity (Flutter, Divergence, etc.)

243. E. G. Broadbent, "The estimation of wing divergence speeds," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2288, Dec. 1945 (issued in 1948), pp. 1-10.

A method is presented for calculating the subsonic divergence speed for a straight tapered wing. The wing is considered elastic and the evaluations are made for a wing in which the torsional stiffness varies as the fourth power of the chord. The method is also adapted to give an approximate solution for the aileron-reversal speed.

The results for divergence speed and aileron reversal are expressed in simple form and are compared with a simplified criterion for bending-torsion flutter. Curves are presented which facilitate calculation of the divergence speed in terms of taper ratio. It is indicated that taper is beneficial.

M. V. Barton, USA

244. E. A. Krasilshchikova, "Influence of end heads on the oscillating wing motion at supersonic velocity" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Nov. 1, 1947, vol. 58, pp. 543-546.

In continuation of previous investigations [Krasilshchikova, *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, 1947, vol. 11, pp. 147-164; and *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*,

1947, vol. 56, p. 571] the author considers the linearized problem of vibrations of a thin elastic wing of finite span in a supersonic flow. In the previous note the author gave the formula for the potential function under the assumption that the normal components of the velocity along the wing are known. She shows in the present note that the determination of the normal component of the velocity can be reduced to an integral equation of Abel's type. In this and the next note the author describes a procedure for solving this equation.

The solution $\theta(x, y)$ of the equation

$$(*) \int \int \frac{\theta(\xi, \eta) \cos [\lambda \sqrt{(x-\xi)(y-\eta)}]}{\sqrt{(x-\xi)(y-\eta)}} d\xi d\eta = f(x, y)$$

is written in the form of an integral for $\lambda = 0$; and it is shown that the domain in which the potential has to be determined can be subdivided into a number of subdomains (bounded by characteristics) such that the potential in each of these domains can be successively determined provided that the integral equation (*) (which is treated in the author's next note) is solved.

Stefan Bergman, USA

245. E. A. Krasilshchikova, "Influence of end heads on the oscillating wing motion at supersonic velocity" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Nov. 11, 1947, vol. 58, pp. 761-762.

In continuation of the note of the preceding Review, the author determines the function $\theta(x, y)$ from the equation (*) (see preceding Review) where f is a given function. She writes

$$\theta(x, y) = \sum_{n=0}^{\infty} \theta_{2n}(x, y) \lambda^{2n}$$

and obtains a system of recursion formulas, from which the θ_{2n} can be determined.

Stefan Bergman, USA

246. J. Morris and D. Morrison, "The rolling vibration of an aircraft," *Rep. Memo. aero. Res. Coun. Lond.*, no. 2290, Aug. 1945 (issued in 1948), pp. 1-12.

This paper treats the rolling vibration of an aircraft with variation of mass and stiffness of structure taken into account. The method is to divide the aircraft into a system of discrete masses linked by an appropriate system of elastic members. The natural frequencies and modes of vibration of the system are then calculated. The results agree reasonably well with the experimental observation.

Chieh-Chien Chang, USA

247. A. R. Collar, E. G. Broadbent, and Elizabeth B. Puttick, "An elaboration of the criterion for wing torsional stiffness," *Rep. Memo. aero. Res. Coun. Lond.*, no. 2154, Jan. 1946 (issued in 1947), pp. 1-24.

This paper describes briefly the work which led to the adoption in England of a simple criterion for wing flutter, which is very similar to the Küssner formula once used in the United States. The paper puts forward a suggestion for a new nondimensional criterion. The appendix describes an investigation into the effects of taper on flutter speed.

N. O. Myklestad, USA

248. W. G. Molyneux and E. G. Broadbent, "Ground resonance testing of aircraft," *Rep. Memo. aero. Res. Coun. Lond.*, no. 2155, July 1946 (issued in 1947), pp. 1-26.

This report affords a survey of current British practice for the conduct of ground vibration tests of aircraft. The determination of the pertinent natural mode frequencies and shapes is an im-

portant part of accurate flutter analysis, and ground vibration tests are employed to obtain this information experimentally. The present report reviews the equipment and methods recommended for the work by the Royal Aircraft Establishment.

In general, the procedures described by the authors are in agreement with standard American practice, as observed by this reviewer. However, British practice apparently does not take advantage of the use of the elaborate oscillographic equipment developed in this country. The use of this equipment permits the simultaneous recording of numerous data histories, provides a permanent record of the vibration traces, and greatly facilitates accurate vibration analysis.

Martin Goland, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Rev. 211)

249. K. H. W. Thomas, "An introduction to the vortex theory of propellers," *Trans. Inst. mar. Engrs.*, Aug. 1948, vol. 60, pp. 171-176.

After presenting some historical features, the paper covers vortices, circulation, an explanation of lift given to an airfoil, the vortex system for a finite lifting vane, and the application of vortex theory to propeller calculations. The discussion is brief and elementary, with numerous diagrams.

R. C. Binder, USA

250. W. Perl and M. Tucker, "A general representation for axial-flow fans and turbines," *Nat. adv. Comm. Aero. Rep.*, no. 814, 1945 (publ. in 1948), pp. 1-5.

In this highly instructive report the authors show that the ratio of pressure drop to impact pressure for axial-flow fans and turbines can be expressed as a simple function of two variables x and y , where x is the ratio of the relative tangential velocity to the rotor peripheral velocity at the rotor entrance, and y the same ratio at the rotor exit. The relationships between the pressure drop coefficient for the stage, the corresponding coefficient for the rotor, and x and y are plotted in a diagram which permits at a glance to determine the operating conditions for various fan and turbine arrangements of conventional or nonconventional design. Some unusual arrangements are discussed which appear to offer definite advantages in developing fans with high-pressure rise per stage.

The analysis is also extended to include variable axial-flow area through the fan or the turbine. It is shown that for these cases the total pressure (pressure and velocity head) coefficient is a simple function of x and y , and that thus the general diagram includes these cases also with some modification of the definitions.

Karl E. Schoenherr, USA

251. Palmer C. Putnam, "Power from the wind," D. Van Nostrand Co., Inc., New York, 1948. Cloth, 6.2 x 9.25 in., 224 pp., 113 figs, \$6.

This is the story of what might have been a big success—obtaining electrical power from the wind at a time when every kilowatt was in great demand. The project cost a million and a quarter dollars and ended when one of the huge blades broke near the hub. Similar two-bladed "wind turbines" will probably not be constructed soon, since power companies can not afford to invest more than \$125 per kw for the intermittent power of such a 1250-kw windmill, and the cost of design and construction at this time would be about \$190 per kw.

The book is a fascinating one for those interested in power from new sources and will serve as a guidebook for future builders of large and small windmills. There is valuable information on past developments, site selection, aerodynamics, vibrations,

power economics, etc. For example, a method of determining average yearly wind speed from tree deformation is given in Chapter IV. A number of famous engineers and aerodynamicists worked on the design with the author, who was also manager of the project.

The analysis was sound, but the design and construction were hurried because of the desire in 1940 to contribute to power needs of the war. Manufacture and procurement of parts started after a preliminary design, and before final weights were determined. The blade shanks and shank-spar connections were known to be weak when the blades had been designed, but by this time it was too late not only to order new forgings, but even to redesign the blades. In order to use larger spars and shanks it would have been necessary to increase the thickness of the blades. The blades proper, for a radius of 87.5 ft, already weighed 8 tons each and could not well have been made heavier without pyramiding the design problems. We make note of the fact that these blades must have been built like a boxcar, because a helicopter blade for a 40-ft radius can be built to weigh 400 lb or less.

Another error which proved costly was accepting a standard 23.6-in. main bearing, when sturdier ones were indicated. A failure of the inner race of one of these bearings shut the wind turbine down for two critical years.

Despite these and other difficulties a total of 1100 hr of operation was logged in the 3½ years between break-in and the final blade failure in March, 1945. W. C. Johnson, Jr., USA

252. Herbert S. Ribner, "Propellers in yaw," *Nat. adv. Comm. Aero. Rep.*, no. 820, 1945 (issued in 1948), pp. 1-23.

A propeller in yaw develops a side force like a fin. An analysis is presented which incorporates induction effects not adequately covered in previous work, and gives good agreement with experiment over a wide range of operating conditions. The analysis shows that the fin analogy may be extended to the form of the side-force expression and that the effective fin area may be taken as the projected side area of the propeller. The effective aspect ratio is of the order of 8, and the appropriate dynamic pressure is roughly that at the propeller disk as augmented by the inflow.

The analysis also shows that a dual-rotating propeller in yaw develops up to one third more side force than a single-rotating propeller. A yawed single-rotating propeller experiences a pitching moment in addition to the side force. The pitching moment is of the order of the moment produced by a force equal to the side force, acting at the end of a lever arm equal to the propeller radius. This cross coupling between pitch and yaw is small but possibly not negligible. R. C. Binder, USA

253. A. J. R. Lysholm, "A contribution to the solution of the gas-turbine problem," *Proc. Instn. mech. Engrs.*, 1947, vol. 157, no. 36, pp. 498-513.

This paper gives a general survey of the gas-turbine development during the period from 1929 to 1945, and is based upon the ideas and inventions of the author. According to these ideas the gas-turbine improvement was sought in increased machine efficiencies rather than in very high temperatures. Correspondingly the compressor and especially the turbine were laid out with high Parsons figures, that is, small heads per stage.

Most of the proposed schemes first made use of the counter-rotating Ljungström turbine type with reaction blading. The compressor was at first of the multistage radial type with backward bent vanes, with an expected stage efficiency of 80 per cent. Special care was taken to cope with the partial-load control. For single-expansion cycles the turbine should be divided into two turbines operating in parallel, in order to obtain good efficiencies.

For reheat cycles the following layout is proposed: a high-pressure turbine driving the high-pressure compressor, and two low-pressure turbines, one driving the generator and the other driving the low- and intermediate-pressure compressor.

A lightweight plant according to these general ideas was built and tested by Bofors in the period from 1932 to 1935. Difficulties occurred with the sealing glands of the Ljungström-type turbine, and pumping troubles with the compressors were experienced. Later a positive-displacement type of screw compressor was conceived and subsequently built by the Elliott Company, reaching an efficiency of 84 per cent at 2.5 pressure ratio. In the further schemes proposed by the author the Ljungström-type turbine was abandoned in favor of the axial type with higher swallowing capacity. A turbine along these lines was built as a marine unit by Elliott, the test unit giving an efficiency of 29.4 per cent at 2400 hp. E. Haenni, USA

254. L. Malavard, R. Siestrunk, and P. Germain, "Calculation of velocity distributions on the profiles of plane blades (Calcul des répartitions de vitesse sur les profils des grilles planes)," *Rech. aéro. Paris*, July 1948, no. 4, pp. 33-46.

The problem of the velocity distribution in the flow field of a closely spaced plane lattice is solved by combining the method of conformal transformation with the electrolytic analogy of potential flow.

The analogy is used to find experimentally the chord-gap ratio and zero-lift direction of the "equivalent" flat-plate lattice and to determine thereby the transformation from the original lattice, consisting of arbitrary profiles to the equivalent lattice. The transformation from the equivalent lattice to the unit circle completes the solution. A complete numerical example for a lattice of Goettingen 431 R profiles is also given.

Andrew Fejer, USA

Flow and Flight Test Techniques

(See also Revs. 203, 206, 235)

255. J. H. Preston, "Visualisation of boundary-layer flow," *Rep. Memo. aero. Res. Council. Lond.*, no. 2267, Nov. 1946 (issued in 1947), pp. 1-6.

This paper presents a brief history and description of various methods for visualizing boundary-layer flow and indicating transition regions and details. Smoke filaments and gas filaments are discussed, noting that use of the smoke technique is generally limited to low speeds but is particularly useful in the detection of laminar separation. Techniques for determining the turbulent region which are based upon (a) deposition of powders, (b) chemical contamination using active gases which react with the prepared surface and (c) evaporation are also briefly described and discussed. John E. Goldberg, USA

256. Hsue-Shen Tsien, "Wind-tunnel testing problems in superaerodynamics," *J. aero. Sci.*, Oct. 1948, vol. 15, pp. 573-580.

This paper discusses some of the more important problems that may be anticipated in testing the flow of rarefied gases in wind tunnels. The first part deals with the large viscous effects associated with the very low Reynolds numbers, and it is deduced that for the design of the nozzle and test section of the wind tunnel it will not be possible to treat compressibility and viscous effects separately. Experimental data confirm this. Consequently, methods of solving the complete Navier-Stokes equations will need to be developed. The second and third parts deal with the special methods of measuring pressure, density and velocity that are required. For the measurement of pressure Pirani gages are suggested; density can be measured by determining the absorp-

tion of ultraviolet light or the afterglow of nitrogen, and for the measurement of velocity the hot-wire anemometer is commended. The theory of the latter is developed in some detail. The ordinary Pitot tube cannot be expected to be accurate at low Reynolds numbers. The last section discusses the conditions for similarity between model and full-scale tests, and it is shown that complete similarity can only be assured by satisfying a number of exacting conditions. Future research may demonstrate that some of these can be relaxed.

A. D. Young, England

257. A. Balloffet, "Gas flow meters for low Reynolds numbers" (in Spanish), *Cienc. Tecn.*, Aug. 1948, vol. 111, pp. 89-108.

The paper presents the results of a series of tests made with gaseous ammonia on twenty small nozzle and orifice flow meters with diameters between 0.83 and 3.2 mm, diameter of the pipe $d = 11$ mm and Reynolds numbers between 120 and 12,000 (that is, for laminar flow in the pipe). A comparison with the results of previous tests of Peterson, Johansen, and Ambrosius and Spink is given.

Giulio De Marchi, Italy

258. A. de Lathouder and S. I. Wiselius, "The development of the wind-tunnel unit of the National Aeronautical Laboratory" (Dutch with English summary), *Ingenieur 's-Gravenhage*, 1948, vol. 60, Oct. 8, pp. L50-57; Dec. 3, pp. L59-61.

A survey is given of the wind tunnels of the National Aeronautical Laboratory at Amsterdam. In 1940 two wind tunnels were built: (1) a small wind tunnel of cross section 1.5×1.5 sq meters and air velocity 42 to 45 meters per sec; (2) a large wind tunnel 2.1×3 sq meters, 81 meters per sec. Today are planned and under construction: (3) a low-turbulence tunnel 2.1×3 sq meters, 120 meters per sec with a six-component balance and 1:7 model tunnel 0.35×0.43 sq meters, 120 meters per sec; (4) a high-speed tunnel 2.1×3 sq meters, $M = 0.95$ with a six-component balance and 1:5 model tunnel with some variations of the measuring cross section for use as a subsonic and transonic tunnel; (5) a supersonic tunnel 0.4×0.4 sq meters, $M = 5-6$ with schlieren optics and interferometer; (6) a smoke tunnel 1.2×1.8 sq meters, 30 meters per sec for flow observations.

The construction of the wind tunnels (3) to (6) is to be completed in 1952. The article contains many details and sectional plans.

Fritz W. Riegels, Germany

Thermodynamics

(See also Revs. 176, 216, 218, 224, 268)

259. H. Einbinder, "Quantum statistics and the \aleph theorem," *Phys. Rev.*, Oct. 1, 1948, vol. 74, pp. 805-808.

The thermodynamic consequences of the state equation $PV = sE$ (E is energy, s a coefficient) are investigated. For such a system $E = V^{-s}\phi(TV^s)$, $S = \chi(TV^s)$ and $A = E - TS = V^{-s}\psi(TV^s)$. During an adiabatic change TV^s , EV^s , and $PV^{1+1/s}$ are constant. The \aleph theorem is derived: A system of mechanically noninteracting particles in equilibrium in a field-free space which individually obey the laws of non- or completely relativistic quantum mechanics, has $PV = sE$; if $T \rightarrow 0$, then $S \rightarrow C$, a specific constant; and it either has a positive zero-point energy, $E = CV^{-s}$, $(\partial E/\partial V)_T > 0$, or it condenses with $P = CT^{1+1/s}$, $(\partial E/\partial V)_T < 0$.

The theorem is used to predict that a strongly degenerate gas obeying intermediate statistics will possess a positive zero-point energy, $E = CV^{-2/3}$. Further, it clarifies the statement that the third law is a quantum mechanical law, showing that it is satisfied by such a general system. If $PV = sE$ is to be obeyed,

$(\partial E/\partial V)_T \neq 0$; a statistical attraction or repulsion must be present, that is $(\partial E/\partial V)_T$ is greater or less than zero, respectively. Finally, the \aleph theorem yields an almost intuitive demonstration of the Einstein condensation theorem in Bose-Einstein statistics.

Arthur Kantrowitz, USA

260. J. De Boer and R. J. Lunbeck, "The properties of the condensed phase of the light helium isotope" (in English), *Physica Hague*, Dec. 1948, vol. 14, pp. 510-519.

Using the law of corresponding states, corrected for quantum-mechanical effects (see following Review), a number of properties of He^3 are predicted as to order of magnitude. The results should be of interest in connection with the separation of He^3 from the natural isotopic mixture. The critical temperature and pressure are roughly 3.2 to 3.5 K and 0.9 to 1.2 atm respectively. The vapor-pressure formula is

$$\log_{10} p = -1.12/T + 2.5 \log_{10} T + 1.01$$

where p is the vapor pressure in cm Hg and T is the absolute temperature in deg Kelvin. The liquid-vapor equilibrium compositions for dilute solutions of He^3 in He^4 are calculated from the above-quoted data and are found to be in satisfactory agreement with the experimental data.

Serge Gratch, USA

261. J. De Boer and R. J. Lunbeck, "Quantum theory of condensed permanent gases, III. The equation of state of liquids" (in English), *Physica Hague*, Dec. 1948, vol. 14, pp. 520-529.

The equation of state of liquefied permanent gases is discussed from the point of view of the law of corresponding states, corrected for quantum-mechanical effects [see J. De Boer, "Quantum theory of condensed permanent gases, I. The law of corresponding states," *Physica Hague*, 1948, vol. 14, no. 2-3, pp. 139-148]. The method used is basically an extension of the classical Lennard-Jones and Devonshire statistical theory of the liquid phase. As in previous papers by De Boer and co-workers, all quantities are expressed in dimensionless ratios based on parameters of the intermolecular field; the corresponding units are called "molecular units." The quantum-mechanical corrections can be expressed in terms of a single dimensionless ratio. Agreement with experimental data on liquid volumes is satisfactory if a co-ordination number between 9 and 10 is used; only in the case of hydrogen and deuterium are the deviations appreciable.

Serge Gratch, USA

262. Joseph Kaye, "Thermodynamic properties of gas mixtures encountered in gas-turbine and jet-propulsion processes," *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 349-361.

A study of recent data on the specific heats and the ratio of specific heats of combustion products shows that only three tables of the thermal properties are necessary for the calibration of the processes encountered in gas turbines. This is possibly due to the small variation of the thermal properties with fuel composition (hydrogen-carbon ratio). The three tables of properties, each for a selected percentage of theoretical air, are also useful in the solution of problems involving mixtures of air and octane vapor or air and water vapor.

C. A. Meyer, USA

263. John C. Fisher, "The fracture of liquids," *J. appl. Phys.*, Nov. 1948, vol. 19, pp. 1062-1071.

Liquids can be subjected to large negative pressures producing a metastable state. In the present paper, the author discusses two types of metastable liquids. One group comprises ordinary

liquids such as water, mercury, and ethyl alcohol, another comprises subcooled liquids such as glass and quartz. For the first group vapor bubbles form spontaneously and grow until the pressure of the system rises to the equilibrium vapor pressure. For the second group cracks form spontaneously and grow until the pressure rises to the equilibrium vapor pressure. These facts have been known for many years.

The important contribution of this paper is a means of calculation of the rate of bubble formation and of the fracture stress of glass on a unified theoretical basis. The basic argument used is the theory of nucleation developed previously by the author and other workers. This theory states that the rate of bubble formation is proportional to $\exp(-W_{\max}/kT)$ where W_{\max} is the maximum work required for reversible formation of a bubble in a liquid under negative pressure, k is Boltzmann's constant, and T the absolute temperature. In addition, the proportionality factor can be estimated from the theory of absolute reaction rates; expressions for this quantity are also given.

It is shown that the rate of bubble formation can be calculated, and the negative pressure p_t which gives one bubble or fracture in t seconds is found. This pressure p_t is very nearly independent of time and is proportional to $\sigma^{1.5}$ where σ is the surface tension. The fracture stress of glass is found to be proportional to $(E^2\sigma^2)^{1/4}$ where E is the elastic modulus.

The author's conclusions are best stated in his own words as follows: "Nucleation theory has led to theoretical values for the fracture stresses of liquids and noncrystalline solids that are an order of magnitude smaller than the values calculated from the forces required for the simultaneous separation of all atomic bonds cut by a plane surface. The maximum experimental fracture stress values approach those derived from nucleation theory suggesting that the theoretical fracture strengths are now more nearly correct."

"The frequent occurrence of premature fracture has been associated with the presence of surface irregularities. It is well known that fracture cracks in glass grow most easily from pre-existing surface cracks. In liquids, similarly, vapor bubbles are much more easily nucleated at an irregular solid-liquid interface than in the interior of a liquid, and the fracture strength corresponding to failure at an interface can have any value numerically less than the theoretical strength of the liquid." Joseph Kaye, USA

264. Jean Brodin, "Graphical prediction of the liquid-vapor and liquid-liquid equilibria of mixtures (Prevision graphique des équilibres liquide-vapeur et liquide-liquide des mélanges)," *C. R. Acad. Sci. Paris*, Nov. 22, 1948, vol. 227, pp. 1080-1082.

The author introduces the concept of isodynes, that is, the mixtures for which the ratios of the fugacities of the various components are equal. A plot of the pressure versus the specific volume for constant temperature and constant fugacity ratios is used to represent phase equilibria. H. C. Brinkman, Holland

265. Arthur S. Iberall, "The effective 'Gamma' for isentropic expansions of real gases," *J. appl. Phys.*, Nov. 1948, vol. 19, pp. 997-999.

In an isentropic expansion of a real gas, the relationship $p v^\gamma = C$ (γ , C constant) is not exact. A development is given for an accurate exponent $\bar{\alpha}$ which permits the use of the relationship $p v^{\bar{\alpha}} = p_1 v_1^{\bar{\alpha}}$ for an isentropic expansion between any two pressures p_0 and p_1 . For this purpose, the quantity α is defined by the relation: $\alpha = -(v/p)(\partial p/\partial v)_s = -\gamma(v/p)(\partial p/\partial v)_T$. Then $\bar{\alpha}$ is given by the relation: $\bar{\alpha} = [\ln(v/v_0)]^{-1} \int_{v_0}^v (\alpha dv/v)$.

Formulas for the first few coefficients of two Taylor series for $\bar{\alpha}$ are given, one in terms of $v - v_0$ and the other in terms of $p -$

p_0 . These coefficients may be computed from the initial conditions of the gas, the expansion ratio, equation of state data, and data on the variation of specific heat with temperature. The error introduced by using the specific heat ratio γ instead of $\bar{\alpha}$ is discussed. This error amounts to only a few tenths of one per cent for such a gas as nitrogen at atmospheric pressure and ordinary temperatures, but is appreciably larger at high pressures. The error introduced by using γ instead of α in the calculation of the velocity of sound is of the same order of magnitude.

Serge Gratch, USA

Heat Transfer; Diffusion

(See also Rev. 281)

266. Le Roy Bromley, "Heat transfer in film boiling from horizontal tube," *U. S. atom. energy Comm. Doc.*, no. 1628, Jan. 21, 1947, pp. 1-27.

Equations are presented for the calculation of heat-transfer coefficients in film boiling from horizontal and vertical cylindrical surfaces. The relations are developed theoretically by considering that heat is transferred through the vapor film by the combined mechanism of conduction and radiation. The effects of conduction and radiation are first considered separately by deriving expressions for the heat-transfer coefficients for each condition alone; a relation is then given for the combination of the two effects to obtain the complete heat-transfer coefficient for film boiling. The basic method employed could be applied to derive equations for film boiling from most other shapes.

Experimental data and results are presented for film boiling of water and of nitrogen from a horizontal carbon tube. Heat-transfer coefficients calculated from the derived equations checked the experimental results to within the accuracy of the experiments.

According to the reviewer's calculations a temperature difference of approximately 50 F between the surface and the saturated liquid is sufficient to produce film boiling of most substances. The highest heat flux obtained was about 100,000 Btu per hr per sq ft.

Y. S. Touloukian, USA

267. T. S. Nickerson and G. M. Dusinger, "Heat transfer through thick insulation on cylindrical enclosures," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1948, vol. 70, pp. 903-906.

The well-known "relaxation" method of calculation is used to calculate the heat transfer through relatively thick insulation applied to an enclosure having the form of a short cylinder with flat ends. Due to the fact that the cylinder is short, the corner effects cannot be neglected. The results are plotted in parametric form, the area ratio being plotted against the diameter to length ratio, for various values of the ratio of insulation thickness to cylinder diameter. Use of either the arithmetic-mean area or geometric-mean area in this case is shown to give large errors. A brief discussion is included in the paper.

Alexander Mendelson, USA

268. Pol Duwez and H. L. Wheeler, Jr., "Experimental study of cooling by injection of a fluid through a porous material," *J. aero. Sci.*, Sept. 1948, vol. 15, pp. 509-521.

The high rates of heat transfer encountered in jet propulsion may be coped with by making the hot parts of a porous material, and forcing the cooling fluid through the pores in a direction opposite to that of the heat flow. This method is called "sweat cooling."

Experiments were performed on sweat-cooled cylindrical

specimens of porous copper, nickel and stainless steel, employing hydrogen and nitrogen gas, respectively, as coolants. The hot gas streams were produced by means of an oxygen-hydrogen burner and of a gasoline-air burner.

The results of the experiments are shown in tabulated and in graph form. On a weight-flow basis, hydrogen was five times more effective than nitrogen. An attempt was made to correlate the results of the experiments with available theoretical concepts. The results can be summarized as follows:

1 The surface temperature of the porous material decreases with increasing coolant flow for the same temperature and velocity of the major gas flow.

2 The relationship between the surface temperature and the coolant flow is influenced by the temperatures and velocities of the main gas stream, by the nature of the coolant employed and by the character of the porous material.

3 It is possible to correlate all experimental results obtained by means of a semiempirical relationship which functionally connects the temperatures of the turbulent flow, of the entering coolant and of the sweat-cooled wall with assumed or computed properties of the wall layer of coolant found on the inside of the chamber of the main flow.

4 Only fair agreement was obtained between fundamental theory and experimental results. This appears to be due at least in part to uncertainty concerning the actual experimental conditions.

Hans F. Winterkorn, USA

Theoretical and Experimental Methods

(See also Revs. 157, 158, 159, 213, 279)

269. James Franck, "Remarks about the role of pure science in general education," *Courant Anniv. Vol.*, Interscience Publishers, New York, 1948, pp. 139-144.

The exclusion of science from a general education in the humanities, although understandable on historical grounds, can no longer be justified. During the past century science has profoundly influenced philosophical thought, while that part of education which aims at developing the capacity for independent reasoning and judgment, and which is of the greatest importance today, can probably be more strongly influenced by science than by any other subject. More thought should be given to the problem of teaching the fundamentals of science to students who have no intention of becoming scientists, although the object is clearly to achieve an insight into the method of science rather than the learning of a number of more or less unrelated facts.

A. W. Skempton, England

270. W. J. Duncan, "Technique of the step-by-step integration of ordinary differential equations," *Phil. Mag.*, July 1948, vol 39, pp. 493-509.

The author first postulates that if an ordinary differential equation or set of equations is integrated step by step, the error in the approximate solution at the end of the interval over which the integration is performed is a function of the number n of equal steps into which the interval is divided. This error is expanded in an infinite power series of n^{-1} beginning with the k th power of n^{-1} , and k is defined as the index of the process which is used in the step-by-step integration. Thus, for a Euler process, $k = 1$. If only the leading term in the infinite power series for the error is retained, the error $e_r(t)$ in the value $x_r(t)$ at the end of the interval of integration is

$$e_r(t) \cong n^{-1}e_o$$

where e_o is an undetermined coefficient. If the same process is carried out with two different numbers n_1 and n_2 of intervals, and $x_{r1}(t)$ and $x_{r2}(t)$ are the values obtained at the end of the interval, then

$$e_o \cong x_{r2}(t) - x_{r1}(t)/(n_1^{-k} - n_2^{-k}).$$

The approximate solution to two differential equations involving processes of index 1 and 2 are presented, and the corrected solutions are shown to agree with the exact solutions within 0.01 to 0.03 per cent. The author closes with a very excellent classification of all of the known methods for the numerical integration of ordinary differential equations.

Henry J. Barten, USA

271. N. V. Korolkov and G. K. Koozminok, "Electrical integrator for ordinary linear differential equations with constant coefficients" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekhn. Nauk.)*, Apr. 1948, no. 4, pp. 517-532.

The paper describes an electrical integrator designed according to ideas of L. I. Gutenmacher [*C. R. Acad. Sci. URSS*, 1940, no. 5]. The equations which can be solved on this integrator are of

the type $\sum_{k=1}^n (a_{ik} + b_{ik}D)u_k = F_i(t)$, where $D = d/dt$, a and b are constants and F a given "forcing" function.

The circuit consists essentially of a set of resistances, capacitances, quadripolar amplifiers (that is, amplifiers with unsymmetrical transmission coefficients: $N_{1,2} \neq N_{2,1} \neq 0$), voltage dividers and voltage sources. If the voltages representing the initial values cannot be impressed simultaneously, the authors show what alternative initial voltages should be impressed instead.

The right-hand sides of the equations, if periodic, are given by rotating commutators. To reduce the error caused by the jumps from segment to segment, the integrator is connected to the commutator through a low-pass filter with a low upper limit of the pass band.

A worked-out example of the solution of a system of six differential equations with constant coefficients is given.

A. W. Wundheiler, USA

EDITOR'S NOTE: Abridged from M. Daniloff's review in *Mathematical Reviews*.

272. Hans Lewy, "On the convergence of solutions of difference equations," *Courant Anniv. Vol.*, Interscience Publishers, New York, 1948, pp. 211-214.

The author considers the numerical determination of the solution $\exp[i(\alpha x + \beta y + \gamma z)]$ of the equation $u_{xx} = u_{xx} + u_{yy}$ by the finite-difference procedure. If h, k, l are the mesh dimensions, and (*) $l^{-2} \geq h^{-2} + k^{-2}$, there exists a real value of γ for every choice of α, β . This value can be used for numerical solution of initial-value problems by applying the principle of superposition of plane-wave solutions. Condition (*) assures convergence.

A. W. Wundheiler, USA

273. Menahem Schiffer, "An application of orthonormal functions in the theory of conformal mapping," *Amer. J. Math.*, Jan. 1948, vol. 70, pp. 147-156.

Let D be a domain in the z -plane, bounded by n smooth curves C_i and containing the point at infinity. Let Λ be the class of functions $f(z)$, which are regular in D , L^2 integrable over D , and possessing a uniform integral $F(z)$ in D . The kernel function $K(z, \xi) = \sum f_n(z)f_n(\xi)$ of Λ with respect to D may be constructed from any complete orthonormal system $f_n(z)$ of Λ and is independent of the system chosen. The author extends these

ideas and definitions to a wider class Ω , which may have a singularity at infinity. By constructing the kernel function $k(z, \xi)$ of Ω with respect to D in two ways he obtains a new proof of his inequality $S(D) \geq 2\pi^{-1}E(D)$, where $S(D)$ is the span of the domain and $E(D)$ the area of the complementary domain, and he also obtains expressions for the functions which map D onto slit domains in terms of $K(z, \xi)$ and $k(z, \xi)$.

Courtesy of *Mathematical Reviews* W. K. Hayman, England

Acoustics

274. F. L. H. M. Stumpers, "On the calculation of impulse-noise transients in frequency-modulation receivers" (in English), *Philips Res. Rep.*, Dec. 1947, vol. 2, pp. 468-474.

This article presents a calculation of the effect of impulse-noise transients in a frequency-modulation receiver having as an intermediate frequency filter a single resonant circuit. The series development of the expression for the phase is used. The calculation is carried through by conventional Laplace transform methods. The superiority of a frequency-modulation receiver over an amplitude-modulation receiver with regard to discrimination against noise of this sort is shown. The method of application of the general method to other types of filters is discussed.

R. G. Wilson, USA

Ballistics, Detonics (Explosions)

(See also Rev. 220)

275. G. Lampariello, "On the dynamics of a point of variable mass (Sur la dynamique du point matériel de masse variable)," *C. R. Acad. Sci. Paris*, July 5, 1948, vol. 227, pp. 35-37.

The simplest case of rocket motion (radial direction of motion in the centrally symmetric gravity field of the earth, and no other forces present) lends itself easily to integration by iteration, if the burning rate is constant. Expressions for velocity and distance obtained after the first step are given.

G. Kuerti, USA

276. A. R. Bryant, "The pressures exerted by an underwater explosion bubble on a circular target plate in a disk-like baffle," *Proc. phys. Soc. Lond.*, Oct. 1948, vol. 61, pp. 341-351.

The problem studied in this excellent work is that of a circular deformable target plate mounted in a rigid annular baffle, the whole being immersed in an incompressible fluid; at a certain distance along the axis normal to the target and passing through its center, an explosion is set off. The pulse from the explosion is not shock-fronted, the rise and fall of pressure being gradual, and the effects of this pulse on the target assembly are considered. The normal fluid velocities everywhere on the rigid baffle and behind the target plate are taken to be zero (the latter condition is meant to approximate an air-backed target in underwater explosion tests); the velocities at the front face of the target follow a predetermined deflection mode of the target plate.

Let ϕ_1 be the velocity potential due to a point source of strength 4π located at the explosion point and in the presence of a rigid target plate and baffle; and let ϕ_2 be the velocity potential which arises with the point source absent, due only to the motion of the target plate with unit central velocity. If the explosion bubble has the small radius a and the central velocity of the target is called \dot{Z} , then the velocity potential for the complete system is approximated by $\phi = a^2\dot{a}\phi_1 + \dot{Z}\phi_2$. The velocity potentials ϕ_1 and ϕ_2 are written as infinite series of Legendre functions, and the equations of motion for the fluid-target system are established in Lagrangian terms.

The results of physical interest concern the pressure distribution

over the target plate of various configurations of the system; also, the "virtual mass," which is the increase in effective inertia of the target due to the mass of fluid which is set in motion by it. A detailed numerical analysis is given to show the significance of the results.

Martin Goland, USA

277. Ernest Esclançon, "On the realization of permanent satellites of the earth by means of terrestrial projectiles (Sur la réalisation de satellites permanents de la terre au moyen de projectiles terrestres)," *Mémor. Artill. fr.*, 1947, vol. 21, no. 4, pp. 1007-1019.

The creation of artificial satellites of the earth by jet or rocket propulsion is a distinct possibility. The important point is to be able to propel the rocket beyond what corresponds to the earth's atmosphere. The author shows, under quite simplified ballistic hypotheses, that once the projectile gets beyond the earth's atmosphere, one can write down conditions which the velocity of the projectile must satisfy if it is to follow an elliptic orbit around the earth. The author emphasizes that his results must be considered as in part speculative since there are a number of unknowns at the present time. One of these is the "radius" of the earth's atmosphere which plays a prominent part in his results.

Benjamin Epstein, USA

278. H. Muraour and G. Aunis, "Force and covolume of colloidal powders (La force et le covolume des poudres colloïdales)," *Mémor. Artill. fr.*, 1948, vol. 22, no. 1, pp. 133-170.

This is a detailed presentation of a similar work of which the authors gave a résumé in *C. R. Acad. Sci. Paris*, May 19, 1948, vol. 226, pp. 1588 to 1590 (see also Rev. no. 1424, *APPLIED MECHANICS REVIEWS*, September 1948). A series of experiments were carried out with five different powders in a closed bomb with a volume of 150 cm³. The pressures were recorded with crusher gages. The Noble and Abel equation of state for the powder gases (Van der Waals' equation reduced to its first term) was found to be adequate, and the force (that is, the product RT) and the covolume of these five powders were determined. The heat loss to the walls of the bomb was taken into consideration by assuming it to be proportional to S/V where S is the heat-transfer surface and V is the volume of the bomb. By introducing thin steel sheets, S is increased and the straight line obtained by plotting pressure versus S/V for the same charge is extrapolated. The intercept on the pressure axis is regarded as the true pressure. Curves and tables are given.

Although attention is called to the fact that the method is not accurate for slow powders where the S/V method of correction for heat loss fails, no discussion is given of the accuracy with which pressure measurements with crusher gages can be recorded.

Ahmed D. Kafadar, USA

Soil Mechanics, Seepage

(See also Revs. 197, 208)

279. Jacob Feld, "Early history and bibliography of soil mechanics" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, pp. 1-7.

Fifty early references to soil-mechanics contributions are listed, dated from 1679, and short discussions of these are given as an outline of the origins of soil-mechanics history. Credit for basic ideas is given to Galileo, Lambert and others. French military engineers of the 17th century made empirical and analytical approaches to the problems of earth pressures and earth slopes; Colomb's essay in 1773 changed the entire method of approach to these problems, and stimulated many experimental and analytical investigations. In the early 19th century there was

study of such subjects as shore erosion, bin pressures, flow of soils from orifices, pile foundations, "sand piles," and lateral earth pressures.
C. Martin Duke, USA

280. P. Y. Poloobarinova-Kochina and S. V. Falkovich, "Theory of seepage of fluids in porous mediums" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Nov.-Dec. 1947, vol. 11, pp. 629-674.

This is a comprehensive report on Russian contributions to the theory of seepage of an incompressible fluid through a porous medium. It gives a review of a great number of exact solutions for the corresponding steady and unsteady motion with or without a free surface. The paper contains a seven-page bibliography.

Courtesy of Mathematical Reviews

A. Weinstein, USA

281. T. Edelman, "Problems of soil-settlement" (in English), *Proc. sec. int. Conf. soil Mech. found. Engng.*, 1948, vol. 1, pp. 30-40.

The author proposes a theoretical method for approximating the time-rate of settlement of a ground layer subjected to successive (but different) load increments. The "secular" (secondary) consolidation, which plays a part in this process, is considered to be due to the viscous flow of the "solid" water surrounding each clay particle. The analysis is developed on the assumption that the law governing the secondary consolidation process is entirely analogous to the one that governs hydrodynamic (primary) consolidation.

Gerald A. Leonards, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 205, 229, 231, 251)

282. R. Penndorf, "The ozone content of the middle stratosphere (Der Ozongehalt der mittleren Stratosphäre)," *Z. Met.*, Aug.-Sept. 1947, vol. 1, pp. 345-357.

The vertical distribution of ozone is important with respect to a number of meteorological problems. The ozone content and the upper boundary of the ozone layer are determined from data on absorption at different heights of light of various wave lengths. With horizontal incidence of solar radiation 99 per cent absorption occurs for 2000 Å, 2600 Å, and 3000 Å at the respective depths of 5 km, 20 meters, and 5 km, below the ozone surface. For vertical incidence the respective necessary distances are 25, 12, and 30 km.

From the intensity reduction of the Na-luminescence at dusk,

the height of the ozone layer can be calculated exactly if the reduction is measured at two different vertical angles. The respective theory is developed and methods for measurement are proposed. Using the data of Vegard and Toensberg, the calculated height of the ozone layer is 50 km. Variations can be explained by accurate estimates of the sources of error. It is proven that the Na-luminescence comes from the E-layer which is especially susceptible to ionization during daytime. The Na-luminescence during daylight and dusk is assumed to result from photo-ionization by solar ultraviolet.

Other phenomena also indicate a height of 50 km as the optical upper limit of the ozone layer. The vertical ozone distribution is recalculated, and the integral of the ozone content between 40 and 50 km is shown to agree with the block distribution derived by Goetz from the reversion effect. Daily variation in ozone content at about 50-km height is conjectured; the ozone distribution is calculated for the photoequilibrium during daylight, for dusk and for nighttime. Hans F. Winterkorn, USA

283. Sokrates Rombakis, "The influence of meteorological factors on the topography of the physical sea level (Der Einfluss meteorologischer Faktoren auf die Topographie des physikalischen Meeresniveaus)," *Z. Met.*, Oct. 1948, vol. 2, pp. 300-305.

A homogeneous nonviscous incompressible ocean is subjected to a perturbing pressure fluctuation at the free surface, the form of the perturbation being $\cos(\alpha x - nt) \cos \beta y$. An expression for the magnitude of the fluctuation of sea level is obtained in terms of the parameters descriptive of the size and period of the perturbing pressure. The equations are of course the familiar tidal equations including the Coriolis terms with the following boundary conditions: (1) infinite extent, constant depth; (2) one side bounded (coast effect), constant depth; (3) two sides bounded, constant depth; (4) two sides bounded, parabolic depth.

The perturbing pressure is interpreted as fluctuations of atmospheric pressure, or of the excess of precipitation over evaporation. Because these are not the only meteorological factors influencing the physical sea level, the title is somewhat misleading. For example, surface-wind friction is of great importance in the problem of the physical sea level in the actual ocean, but of course it cannot be treated using the simple tidal equations.

Henry Stommel, USA

Marine Engineering Problems

(See Rev. 249)